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FATIGUE LIFE PREDICTION FOR SIMULTANEOUS STRESS AND STRENGTH VA--ETC(U)
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FATIGUE LIFE PREDICTION FOR SIMULTANEOUS STRESS AND STRENGTH VARIANCES

10 R.G. LAMBERT 15

THIS WORK WAS PERFORMED ON NAVY CONTRACT N00019-78-C-0407

11 MAR [redacted] 80 12 79

GENERAL ELECTRIC COMPANY
AEROSPACE ELECTRONIC SYSTEMS DEPARTMENT
UTICA, NEW YORK 13503

GENERAL ELECTRIC

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402456

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A103 213	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Fatique Life Prediction for Simultaneous Stress and Strength Variances		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. G. Lambert		8. CONTRACT OR GRANT NUMBER(s) Navy Contract N00019-78-C-0407
9. PERFORMING ORGANIZATION NAME AND ADDRESS General Electric Company Aerospace Electronic Systems Department Utica, New York 13503		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Electronic Systems Command Washington, D.C. 20360 Attn: Code 8131		12. REPORT DATE March 1980
		13. NUMBER OF PAGES 72
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution Unlimited. Approved for Public Release.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

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FATIGUE LIFE PREDICTION

FOR SIMULTANEOUS

STRESS AND STRENGTH VARIANCES

FATIGUE LIFE PREDICTION FOR SIMULTANEOUS STRESS AND STRENGTH VARIANCES

Simple closed form expressions have been found to accurately predict the fatigue life of structures subjected to sinusoidal or random stresses where the applied stress and the material's strength are simultaneous random variables. With appropriate parameter value changes the same equations accurately apply to both the low cycle (inelastic) and high cycle (elastic) fatigue regions. These equations are in familiar engineering terms. Comparisons between analytical predictions and empirical results have shown to be good whenever such comparisons were made.

INTRODUCTION

Many closed form analytical expressions have previously been derived to predict structural fatigue life and mechanical reliability for both sinusoidally and randomly applied stresses and strains {1} {2} {3} {4} {5}. These expressions have been shown to be simple, practical and accurate. They apply to single and multi-degeee-of-freedom systems, to single level or step-stress load situations, and to both low and high cycle fatigue regions. Fracture Mechanics effects are included. In all of these cases the stress/strength and strain/ductility parameters were treated as random variables independently, not simultaneously.

In most practical cases the stress/strength parameters are simultaneous random variables. Stresses vary from part to part and subassembly to subassembly due to dimensional and geometrical differences between parts, fabrication and assembly variances, and structural damping and stiffness variances of adjacent structures. Strengths vary because materials' fatigue curves are a scatterband of failure points, not single lines.

APPROACH SUMMARY

An attempt to rigorously derive a fatigue life expression with the stress/strength parameters treated as simultaneous random variables was unsuccessful in that the final expression was exceedingly complex. Therefore a different approach was evaluated. This approach modified the variable strength fatigue life expression {1} by adding the stress (δ) and strength (Δ) standard deviations in the mean-square sense and substituting the resulting standard deviation ($\Psi = \sqrt{\Delta^2 + \delta^2}$) in place of the strength standard deviation term (Δ). The reasoning behing this approach was as follows: Fatigue failure occurs when stress exceeds strength regardless of whether the stress is "too high" or the strength is "too low". Both deviations from nominal cause a reduction in fatigue life. Since the standard deviations of stress and strength are independent of each other, they should be added in the mean-square sense. This approach, as judged by Monte-Carlo simulation techniques, gives somewhat accurate results but not as accurate as hoped for.

Accuracy was improved by multiplying the stress standard deviation (δ) by the term $(2N_m)^{2/\beta}$. N_m is the median stress cycles to failure. It is the fatigue life if the analysis is done deterministically (i.e. if Δ and δ are zero). β is the slope parameter of the materials "S-N" fatigue curve. This term made the entire expression almost identical to the rigorously derived equation for the case of $\Delta = 0$.

Accuracy was further improved in the region of early fatigue failures by subtracting the term $\dfrac{(2N_m)^{1/\beta}\Delta\delta}{\sqrt{2\beta - \pi/\beta}}$.

APPROACH SUMMARY (Cont'd)

The portion $\sqrt{2\beta - \pi/\beta}$ was required to provide accuracy for both the low and high cycle fatigue regions and for brittle and ductile materials. This worsened the accuracy in the region of the late failures. The above term needed to be added instead of subtracted in that region (i.e. a sign change for $N > N_m$). This worsened the accuracy in the middle failure region. The multiplying term $\S = 2 \text{ erf} \left[20\left(\frac{N}{N_m} - 1\right)\right]$ restored accuracy to all failure regions.

The resultant standard deviation term is

$$\Psi = \sqrt{\Delta^2 + (2N_m)^{2/\beta}\ \delta^2 + \S\ \frac{(2N_m)^{1/\beta}\Delta\delta}{\sqrt{2\beta - \pi/\beta}}}$$

Accuracy of the above expressions was judged by comparison to Monte-Carlo simulation results. The Monte Carlo simulation technique had its accuracy and practicality checked by comparing its results with those known to be theoretically correct and with available empirical results.

Fatigue life is expressed in terms of probability of failure as a function of applied stress cycles and both average and minimum cycles to first failure. For the most part data is presented in the form of histograms of cycles to failure because of the histogram's sensitivity to differences between theoretical and tallied results.

SUMMARY OF RESULTS

The single expression for Ψ provides accurate fatigue life results for ductile and brittle materials, over the early and late failure regions ofr both low (inelastic) and high (elastic) cycle fatigue situations. All of the fatigue life and mechanical reliability equations in references through {5} that originally applied to cases where strength alone was the random variable, can be used for simultaneous stress/strength variances by substituting Ψ for Δ.

The Monte Carlo simulation technique was judged to be both accurate and practical due to good comparisons with results known to be theoretically correct and with available empirical results.

FATIGUE CURVE REPRESENTATION

Modern fatigue curve representation is as shown in figures 1 and 2 {6} {7} Figure 1 is a plot of "true" stress amplitude versus reversals to failure for 1020 HR steel. It covers both the low cycle (plastic or inelastic) and the high cycle (elastic) fatigue regions. The fatigue curve is a single straight line. "True" stress is defined as the applied load divided by the actual cross-section area, which becomes less than the original area as the load is increased. "True" stress is contrasted with "engineering" stress which is defined as the applied load divided by the original cross-section area. Life is in terms of reversals to failure or twice the cycles to failures N_f; there being two reversals for each stress cycle. The fatigue strength coefficient σ'_f can be thought of as being approximately equal to the "true" ultimate strength of the material. The fatigue strength exponent can be thought of as a slope parameter.

Figure 2 is a plot of the same failure data as figure 1 except the ordinate is expressed as "true" strain amplitude. The strain-life curve is the sum of the plastic and elastic strain-life curves. E is the modulus of elasticity. ε'_f is the fatigue ductility coefficient. It can be thought of as a measure of the material's ductility. The fatigue ductility exponent c has a value of approximately -0.5 for most structural materials. The fatigue strength exponent b takes on values of approximately -0.1 for ductile materials to -0.05 for brittle materials. Fatigue curve data for many materials is found in reference {7} . The strain-life curve of figure 2 shows that the plastic strain-life predominates below approximately 10^5 cycles; whereas the elastic strain-life curve predominates above 10^5 cycles. The transition cycles varies widely for different materials.

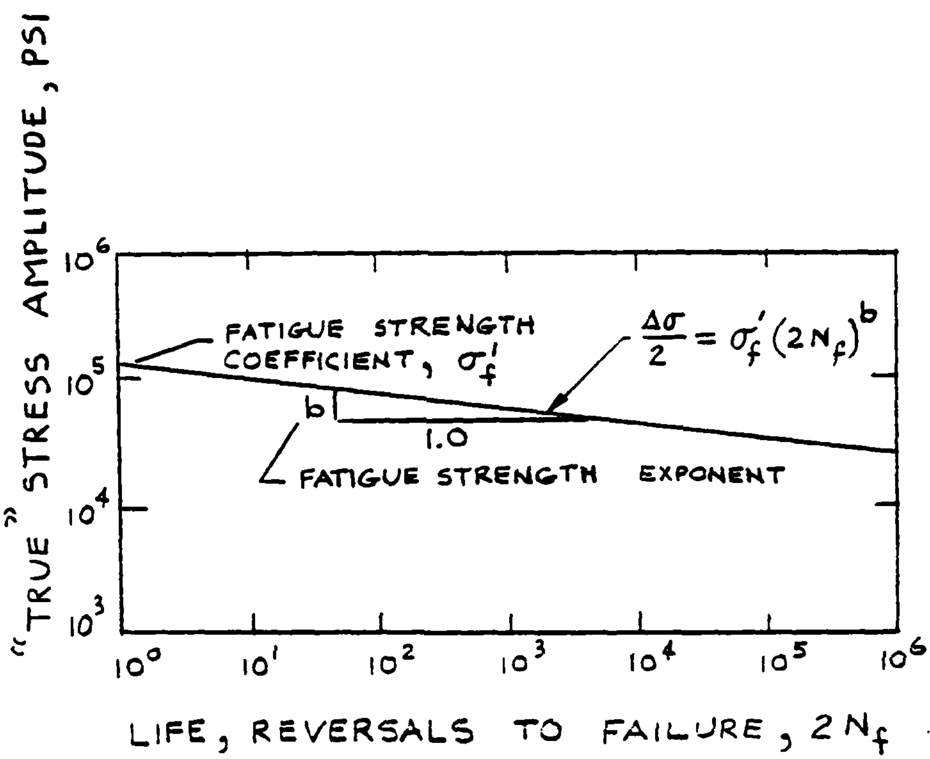


FIGURE 1 STRESS AMPLITUDE VERSUS REVERSALS TO FAILURE, 1020 HR STEEL

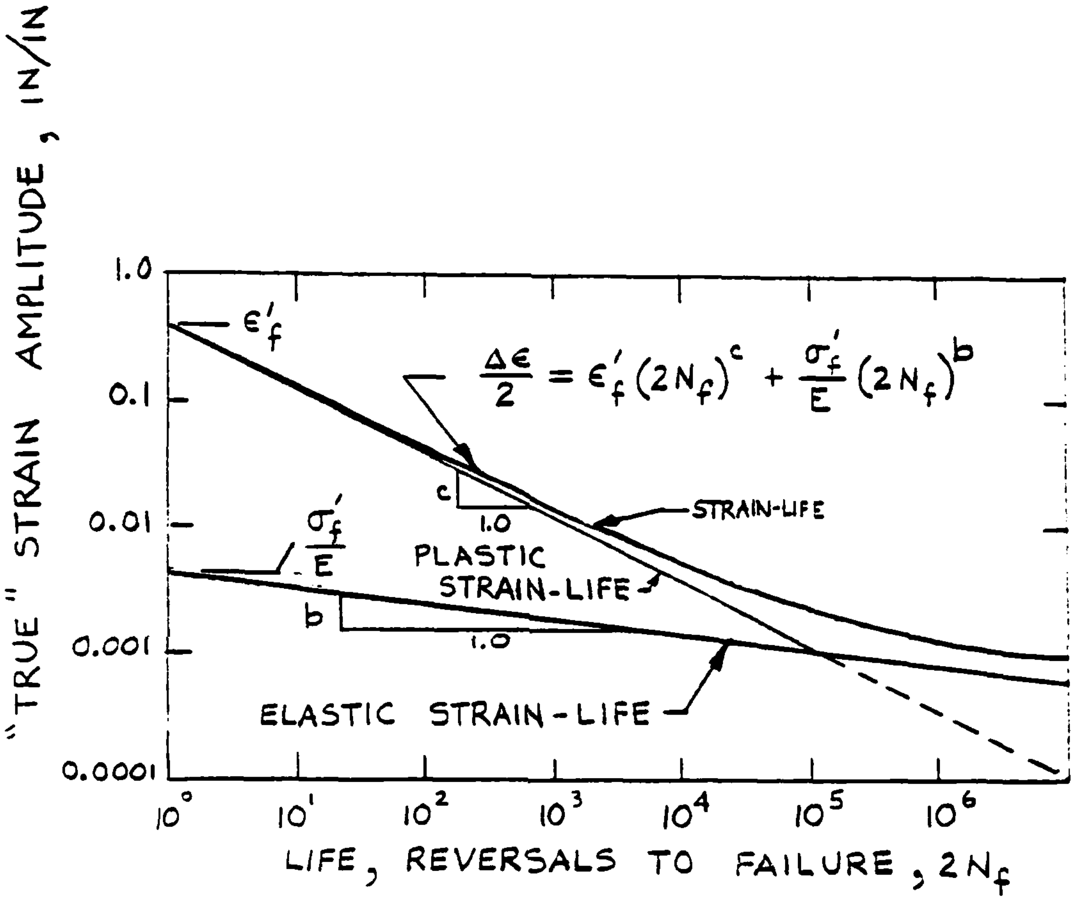


FIGURE 2 STRAIN AMPLITUDE VERSUS REVERSALS TO FAILURE, 1020 HR STEEL

Figure 3 is the plastic strain-life curve of figure 2. Figure 4 is the elastic strain-life curve of figure 2. For the analyses of this paper the strain-life curves similar to those of figures 3 and 4 will be used for various materials. The general form to be used is shown in Figure 5. In the plastic region $\beta \approx 2$ for most structural materials, $\varepsilon_\mu = \varepsilon'_f$ and $\varepsilon = \Delta\varepsilon_p/2$ (compare with figure 3). In the elastic region $\beta \approx 8$ for ductile materials and $\beta \approx 20$ for very brittle materials, $\varepsilon_\mu = \sigma'_f/E$ and $\varepsilon = \Delta\varepsilon_e/2$ (compare with figure 4.) The curve of Figure 5 is expressed as

$$N_f = \frac{1}{2}\left(\frac{\varepsilon_\mu}{\varepsilon}\right)^\beta \tag{1}$$

where N_f = cycles to failure

ε = applied strain amplitude, inches/inch

ε_μ = "y-intercept", in/in

ε_μ represents the material's ductility in the plastic region and the material's strength in the elastic region. Equation (1) and the single line of figure 5 represent a deterministic fatigue curve. Actual fatigue curves are scatter-bands of failure points. The single line represents the median. The scatter-band of points can be represented by letting ε_μ in equation (1) become a Gaussian random variable with mean value $\bar{\varepsilon}_\mu$ and standard deviation Δ_ε. A random variable applied strain amplitude can also be represented as a Gaussian random variable with mean value $\bar{\varepsilon}$ and standard deviation δ_ε. Equation (1) then becomes

$$N_m = \frac{1}{2}\left(\frac{\bar{\varepsilon}_\mu}{\bar{\varepsilon}}\right)^\beta \tag{2}$$

where N_m = median cycles to failure

$\bar{\varepsilon}_\mu$ = average value of ε_μ

$\bar{\varepsilon}$ = average value of ε

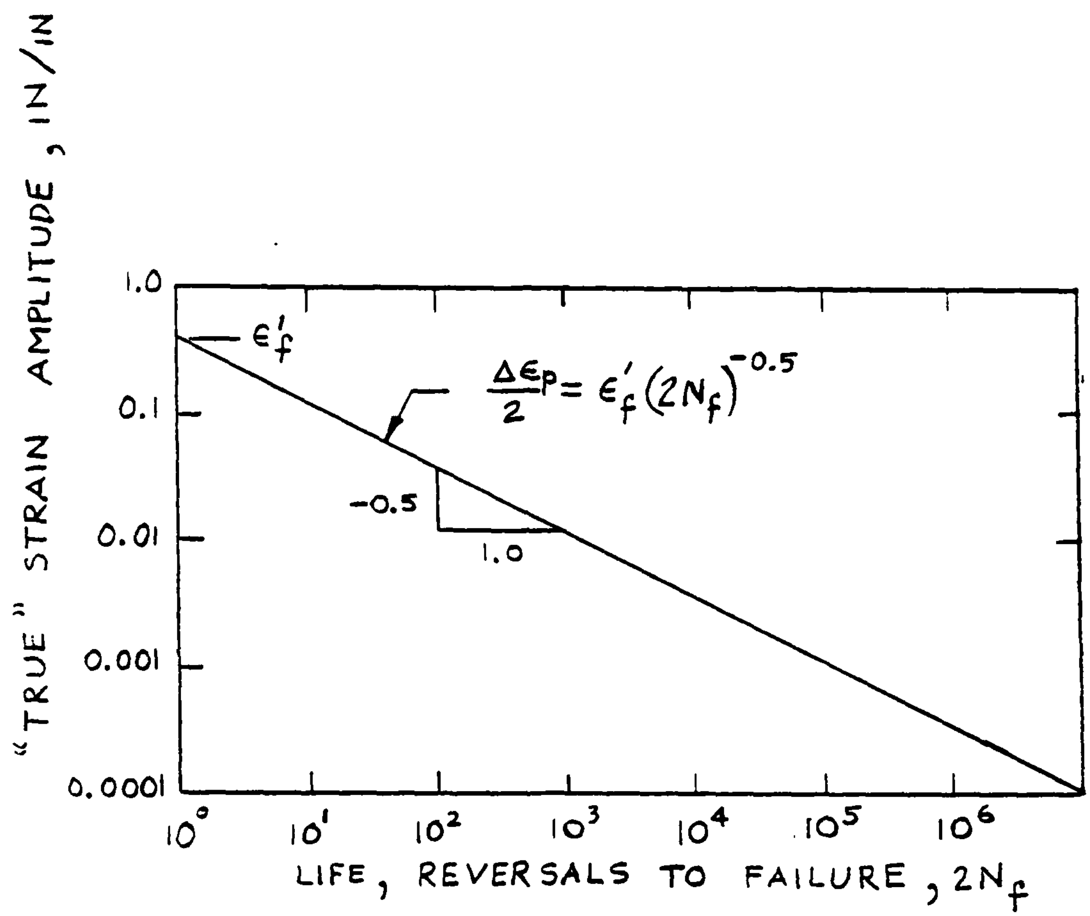


FIGURE 3 PLASTIC STRAIN AMPLITUDE VERSUS REVERSALS TO FAILURE, 1020 HR STEEL

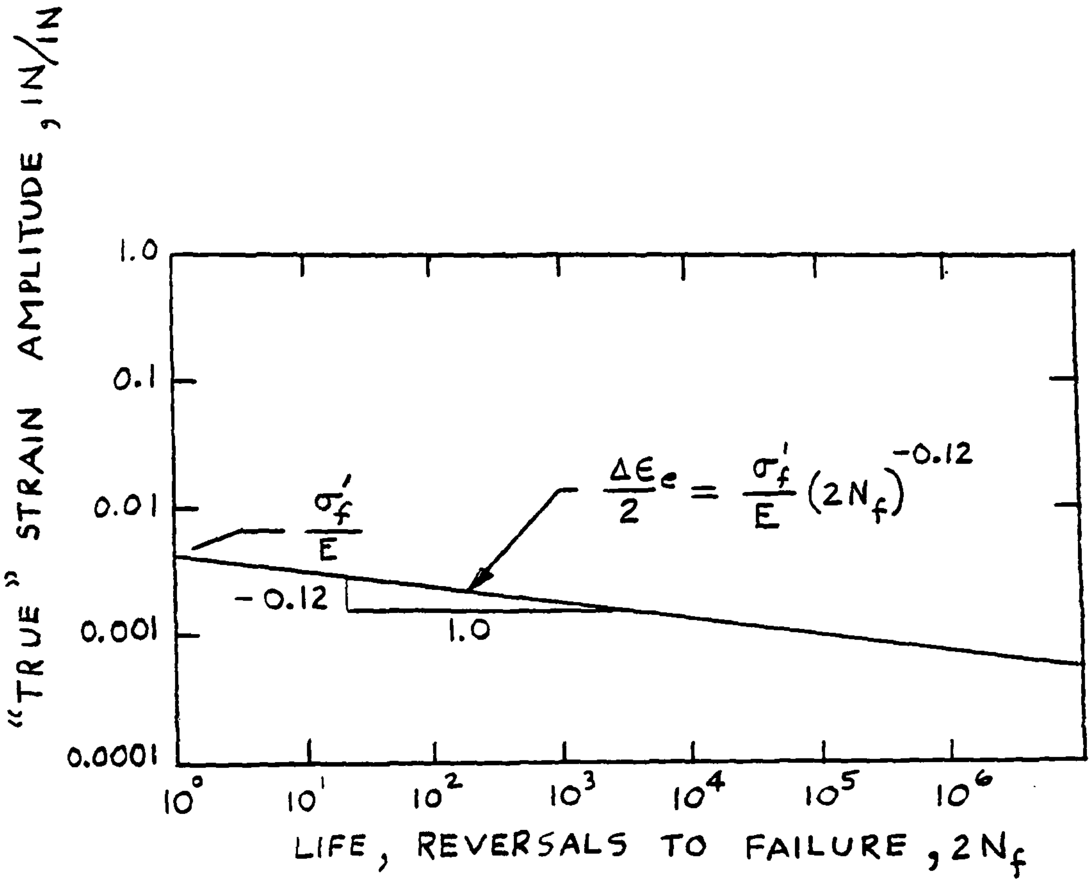


FIGURE 4 ELASTIC STRAIN AMPLITUDE VERSUS REVERSALS TO FAILURE, 1020 HR STEEL

PARAMETER	LOW CYCLE REGION	HIGH CYCLE REGION
β	$-1/c$	$-1/b$
$\bar{\epsilon}_u$	ϵ_f'	$\sigma_f'/E = \frac{2^{1/\beta}\bar{A}}{E}$
$\bar{\epsilon}$	$\Delta\epsilon_p/2$	$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\sigma}{2E}$

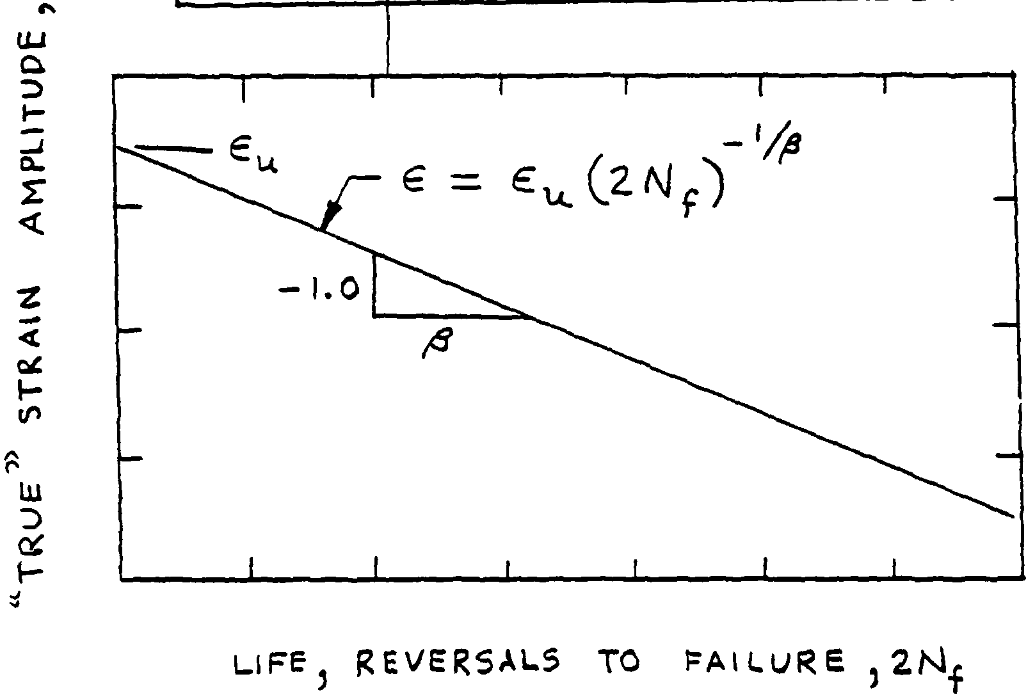


FIGURE 5 GENERAL FORM FOR STRAIN-LIFE FATIGUE CURVES

Equation (2) can equivalently be expressed as follows:

In the low cycle region:

$$N_m = \frac{1}{2}\left(\frac{\varepsilon'_f}{\Delta\varepsilon_p/2}\right)^2 \tag{3}$$

In the high cycle region:

$$N_m = \frac{1}{2}\left(\frac{\sigma'_f/E}{\Delta\varepsilon_{e}/2}\right)^{\beta} \quad ; \ \sigma'_f = \bar{A}\, 2^{1/\beta} \tag{4}$$

$$\frac{\Delta\varepsilon_e}{2} = \frac{\Delta\sigma}{2E} \tag{5}$$

where $\frac{\Delta\sigma}{2}$ = applied stress amplitude, psi

$$N_m = \frac{1}{2}\left(\frac{\sigma'_f}{\Delta\sigma/2}\right)^{\beta} \tag{6}$$

For a random applied stress of rms value $\bar{\sigma}$, define {1}

$$\bar{C} = \left[\frac{2^{-1/\beta}\sigma'_f}{\sqrt{2}}\right]\left[\frac{1}{\Gamma\left(\frac{2+\beta}{2}\right)}\right]^{1/\beta} \tag{7}$$

$$N_m = \left(\frac{\bar{C}}{\bar{\sigma}}\right)^{\beta} \tag{8}$$

The standard deviations associated with strain-life fatigue curves are Δ_ε and δ_ε for the material's ductility and applied strain amplitude respectively. These are in strain units of inches per inch. The corresponding standard deviations associated with stress-life fatigue curves are as follows:

$$\Delta = E\,\Delta_\varepsilon \tag{9}$$

$$\delta = E\,\delta_\varepsilon \tag{10}$$

where E = modulus of elasticity, psi

Δ, δ ∿ psi

ANALYTICAL DERIVATION

The derivation of the fatigue life expressions begins with the derivations of equations for the probability density function of cycles to failures $p(N_f)$ and the probability of failure at N applied stress cycles $F(N)$. Appendix A describes the derivation of $p(N_f)$ simultaneous variations in ε_μ and ε. It is

$$p(N_f) = \frac{2^{1/\beta} N_f^{1/\beta - 1}}{\beta \Delta_\varepsilon \delta_\varepsilon \pi} \left[\frac{1}{2} \sqrt{\frac{\pi}{r}} e^{\frac{-(h^2 - rv)}{r}} \left\{ 2 \bar{\varepsilon} \operatorname{erf}(\alpha_1) + \frac{1}{\pi} e^{-h^2/r} - \frac{2h}{\sqrt{r}} \operatorname{erf}(\alpha_2) \right\} \right] \tag{11}$$

where the variables α_1, α_2, h, r and v are complicated functions of N_f.

$F(N)$ = Probability that $N_f > N$

$$F(N) = \int_0^N p(N_f) dN_f \tag{12}$$

It can be seen by examining equations (11) and (12) that finding a simple closed form expression for F(N) does not appear likely. Without a simple expression for F(N) the derivation of the average cycles to first failure $\bar{N}_1$ and the minimum cycles to first failure $N_{1_{MIN}}$ cannot proceed.

SIMULATION TECHNIQUE

A Monte Carlo technique was used as the simulation method for judging the accuracy of the proposed fatigue life expressions. From equation (1)

$$N_f = \frac{1}{2} \frac{\varepsilon_\mu^\beta}{\varepsilon^\beta} \qquad (1)$$

A sample of the random variable N_f is generated by generating a sample of ε_μ and ε; then performing the operation indicated by equation (1). Each sample of ε_μ is drawn from a Gaussian distribution of mean value $\bar{\varepsilon}_\mu$ and standard deviation Δ_ε. Each sample of ε is similarly drawn from a Gaussian distribution of mean $\bar{\varepsilon}$ and standard deviation δ_ε. Negative values of ε_μ and ε are discarded. The samples of N_f are sorted and stored in array bins according to the sample's value. The quantity of N_f samples that fall into each bin is summed and stored. A printout of the quantity of samples in each bin of the array represents a histogram of N_f for specific values of $\bar{\varepsilon}_\mu$, Δ_ε, $\bar{\varepsilon}$, δ_ε and β.

COMPARISON OF SIMULATION AND THEORETICAL RESULTS

Theoretical results are obtained as follows:

Define $N8 = \varepsilon_\mu^{\ \beta}$ (13)

N8 is the numerator of equation (1). Refer to equation (A-2) in Appendix A.

$N8 = x$

Therefore

$$p(N8) = \frac{(N8)^{1/\beta - 1}}{\beta\ \Delta_\varepsilon \sqrt{2\pi}} \exp\left[- \frac{\left\{ (N8)^{1/\beta} - \bar{\varepsilon}_\mu \right\}^2}{2\ \Delta_\varepsilon^{\ 2}} \right] \qquad (14)$$

Figure 6 shows a graphic illustration of the ε_μ - N8 mathematical transformation. This illustrates the reason for the N8 histogram shape.

$$F(N) = \int_0^N p(N8)dN8 \qquad (15)$$

$$F(N) = 0.5 + \mathrm{erf}\left[\frac{N^{1/\beta} - \bar{\varepsilon}_\mu}{\Delta_\varepsilon} \right] \qquad (16)$$

A histogram array bin quantity q for a bin that extends from N_a to N_b is

$$q = \left\{ F(N_b) - F(N_a) \right\} S \qquad (17)$$

where S = total N8 sample size

Table I shows the values of the parameters for several N8 histograms. A wide range of β values was chosen.

Refer to equation (14). Note that a value of $\beta = 1$ should give a Gaussian histogram (i.e. Case $\boxed{3}'$). Figures 7, 8 and 9 show good agreement between equation (17) and the program generated histogram.

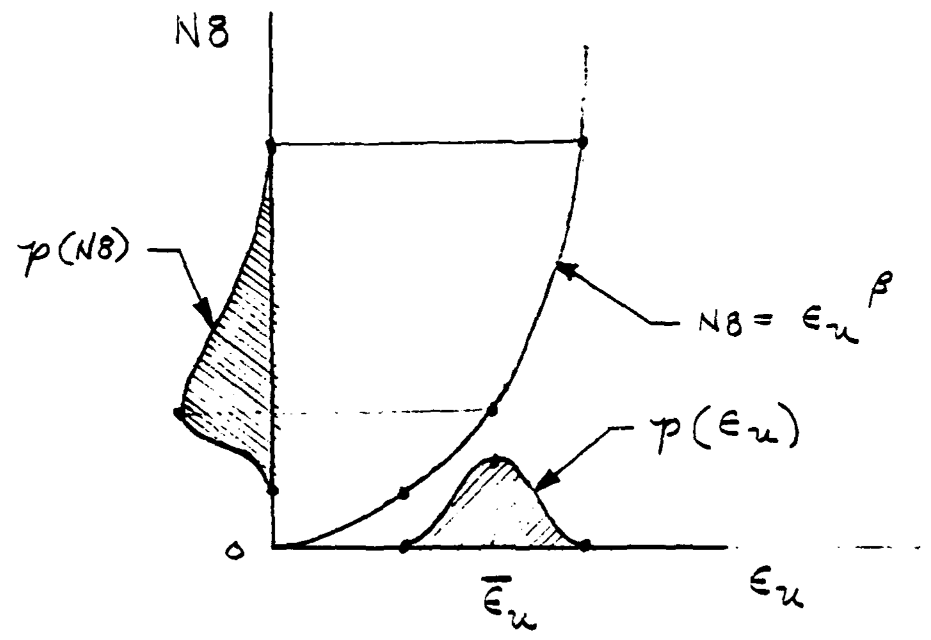


FIGURE 6 GRAPHIC ILLUSTRATION OF ϵ_u-N8 TRANSFORMATION

TABLE I N8 HISTOGRAM DATA

CASE	$\overline{\varepsilon_\mu}$	Δ_ε	δ_ε	$\overline{\varepsilon}$	β	$\overline{N8}$
$\boxed{8}'$	0.2	0.3	0.00079	0.00632	2	0.041
4'	0.0185	0.000925	0.0001726	0.0034515	9.6	2.5E-17
5'	0.0185	0.000925	0.004387	0.00022	12.1	1.3E-21
$\boxed{3}'$	0.2	0.03	0	0.0024	1	0.2

$\overline{\varepsilon_\mu}$, Δ_ε , δ_ε , $\overline{\varepsilon}$, $\overline{N8}$, $\sim$ IN/IN

FIGURE 7 HISTOGRAM OF N8 = ϵ_u

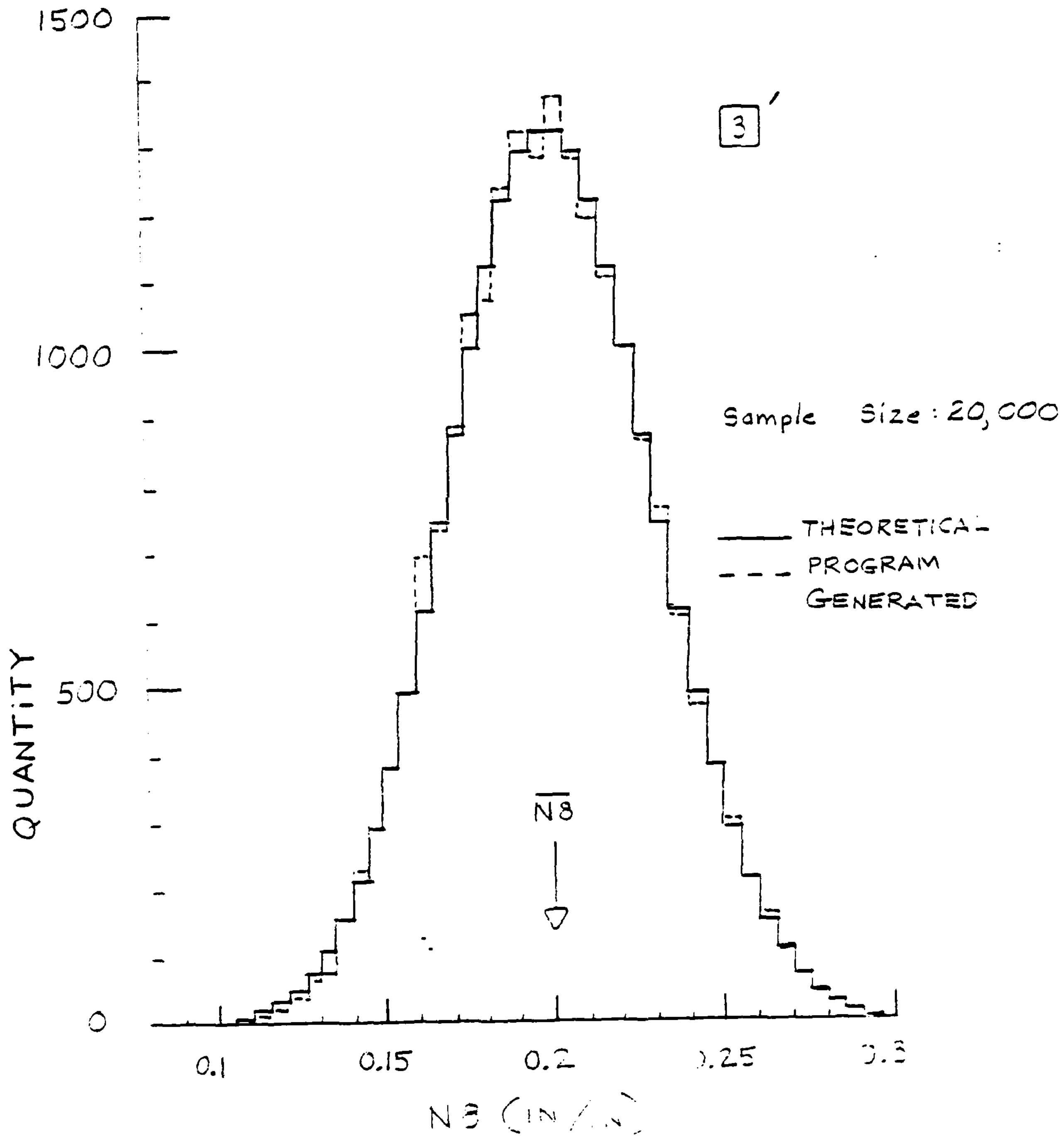


FIGURE 8 HISTOGRAM OF PROGRAM RANDOM NUMBER GENERATOR LEFT SIDE

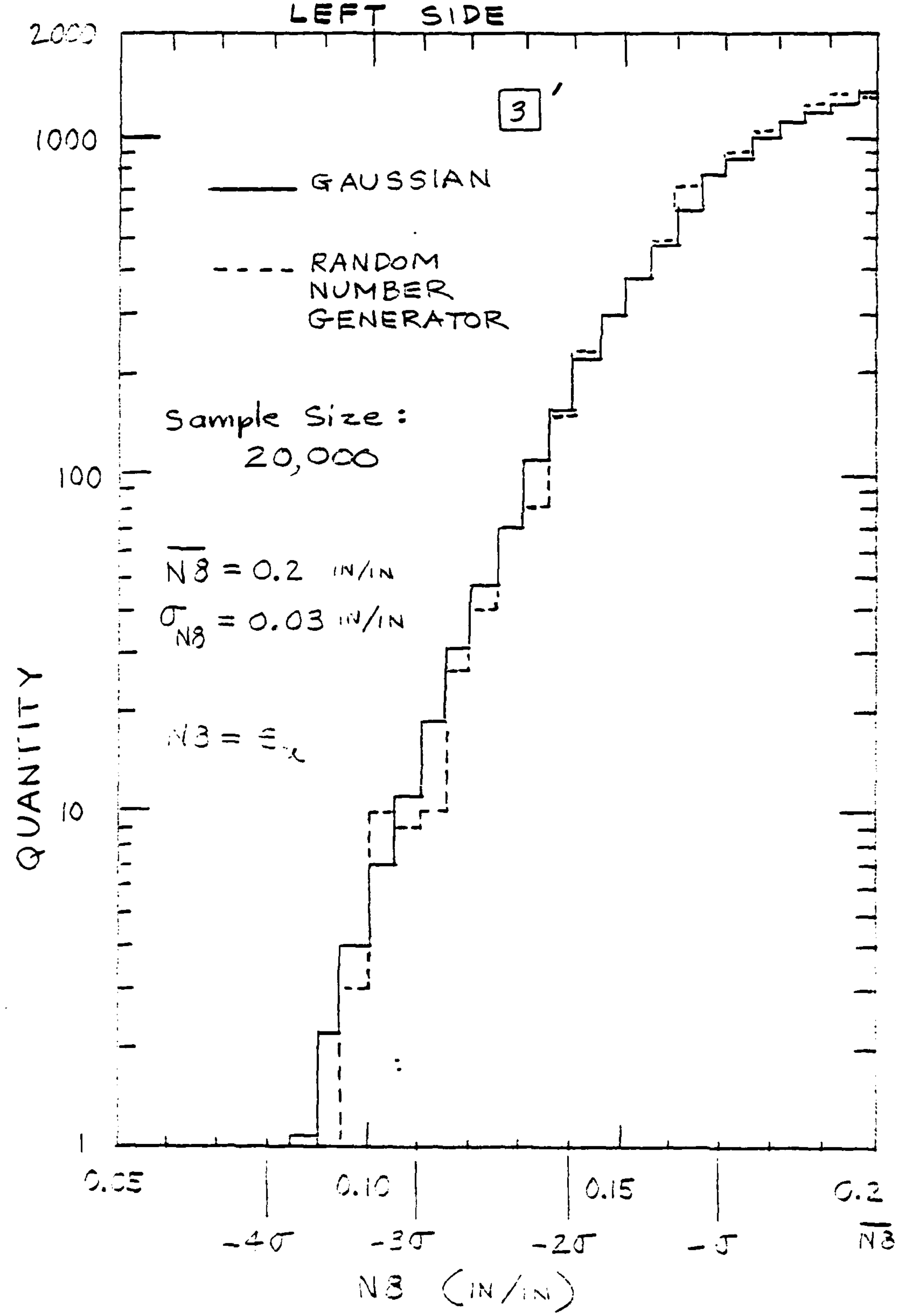
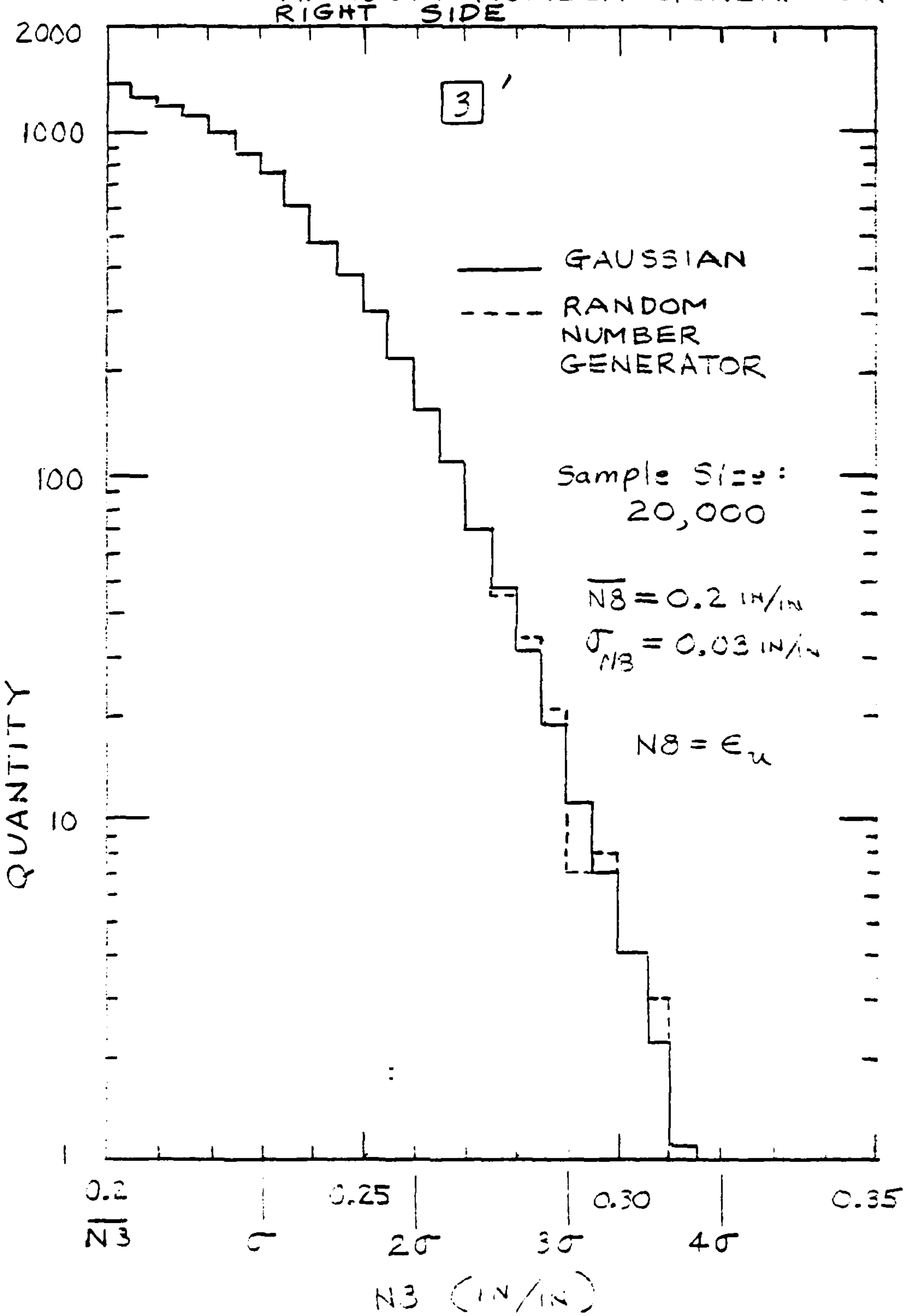


FIGURE 9 HISTOGRAM OF PROGRAM RANDOM NUMBER GENERATOR
RIGHT SIDE



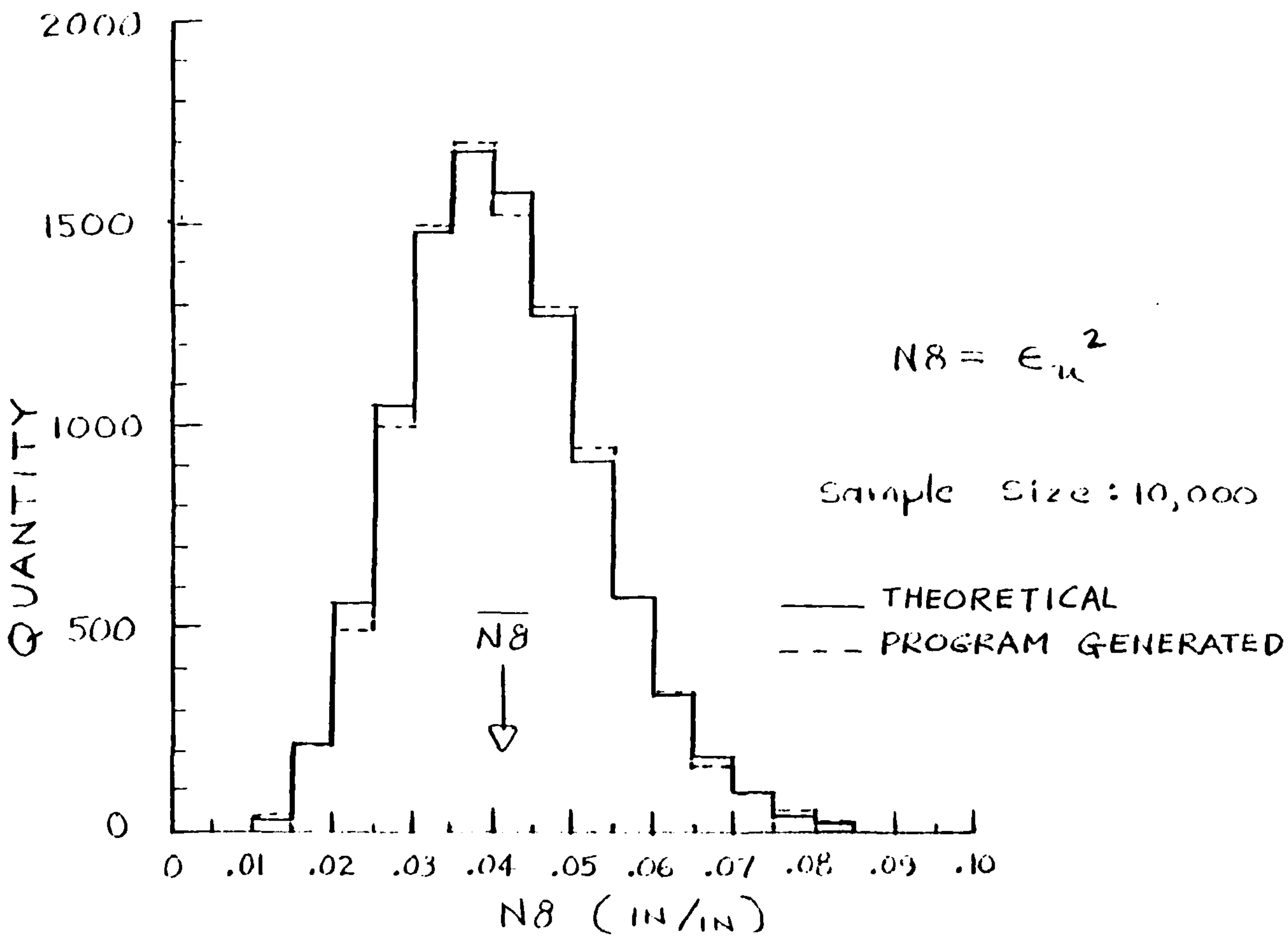
Figures 10, 11 and 12 also show good agreement between theoretical and Monte Carlo results. These curves also show the expected skewing effect of large β values.

Similar results would be obtained for generating samples for ε^{β}, the demoniator of equation (1).

In later sections comparisons will be made between Monte Carlo results and theoretical ones for N_f where ε_{μ} and ε are random variables independently.

In all cases it will be seen that the Monte Carlo simulation technique is accurate compared to theoretical equations.

FIGURE 10 HISTOGRAM OF CASE $\boxed{8}'$



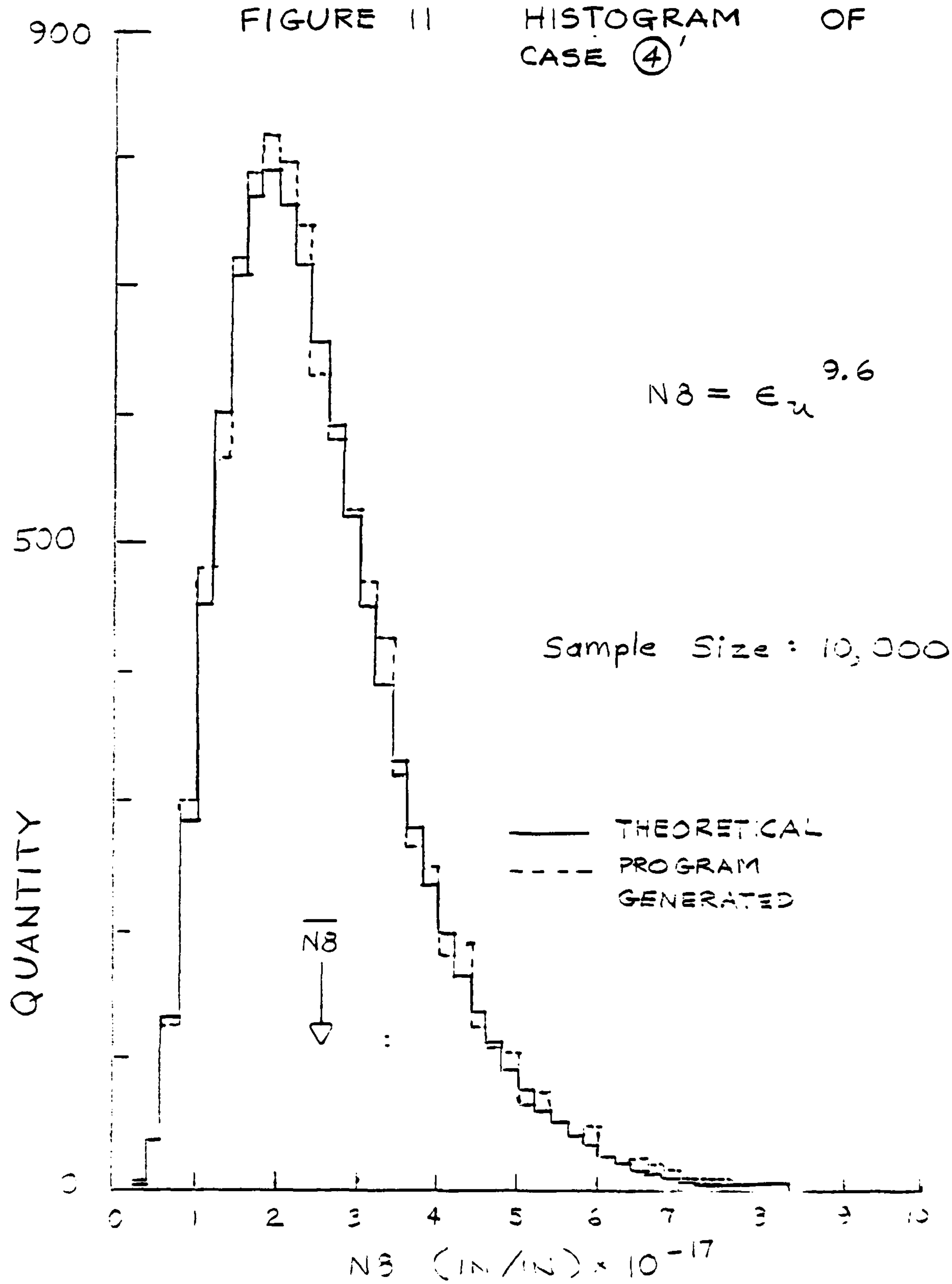


FIGURE 11 HISTOGRAM OF CASE 4
900
500
0
QUANTITY
N8 = ε_u^9.6
Sample Size : 10,000
THEORETICAL
PROGRAM GENERATED
N8
0 1 2 3 4 5 6 7 8 9 10
N8 (IN/IN) × 10^-17

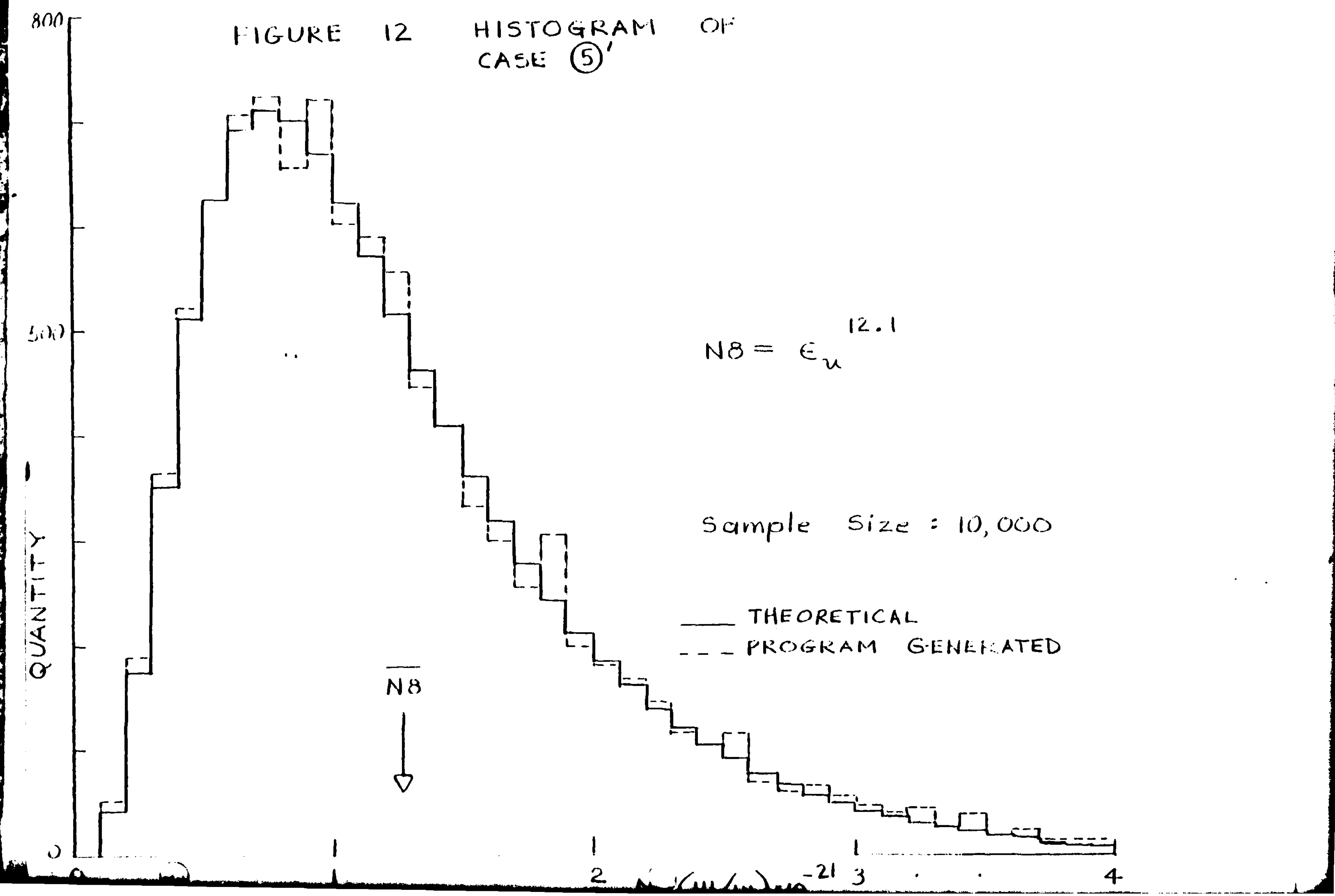


FIGURE 12 HISTOGRAM OF
CASE (5)'
N8 = $\epsilon_u^{12.1}$
Sample Size : 10,000
THEORETICAL
PROGRAM GENERATED
$\overline{N8}$
QUANTITY
800
500
0
2
3
4

PROPOSED FATIGUE LIFE EXPRESSIONS

Equations (2) through (10)are proposed to calculate the median cycles to failure N_m in terms of either stress/strength or strain/ductility parameters. Equations (9) and (10) relate the standard deviations of stress and strain. These equations are to be used in the following proposed fatigue life expressions:

F(N) = Probability of failing at N applied stress cycles

$$F(N) = 0.5 + \mathrm{erf}\left[\frac{\bar{\varepsilon}_\mu}{\Psi_\varepsilon}\left\{\left(\frac{N}{N_m}\right)^{1/\beta} - 1\right\}\right] \tag{13}$$

$$\mathrm{erf}\ (\alpha) = \frac{1}{\sqrt{2\pi}}\int_0^\alpha e^{-y^2/2}dy \tag{14}$$

$$\Psi_\varepsilon = \sqrt{\Delta_\varepsilon^{\,2} + (2N_m)^{2/\beta}\,\delta_\varepsilon^{\,2} + \frac{\S\ (2N_m)^{1/\beta}\,\Delta_\varepsilon\,\delta_\varepsilon}{\sqrt{2\beta - \pi/\beta}}}\ \text{in/in} \tag{15}$$

$$\S = 2\ \mathrm{erf}\left[20\left(\frac{N}{N_m} - 1\right)\right] \tag{16}$$

Figure 13 is a plot of § versus N/N_m.

$\bar{N}_1$ = average cycles to first failure

$$\bar{N}_1 = N_m\left[1 - \frac{3.7195451}{(\bar{\varepsilon}_\mu/\ \Psi_\varepsilon)}\right]^{\beta} \text{cycles} \tag{17}$$

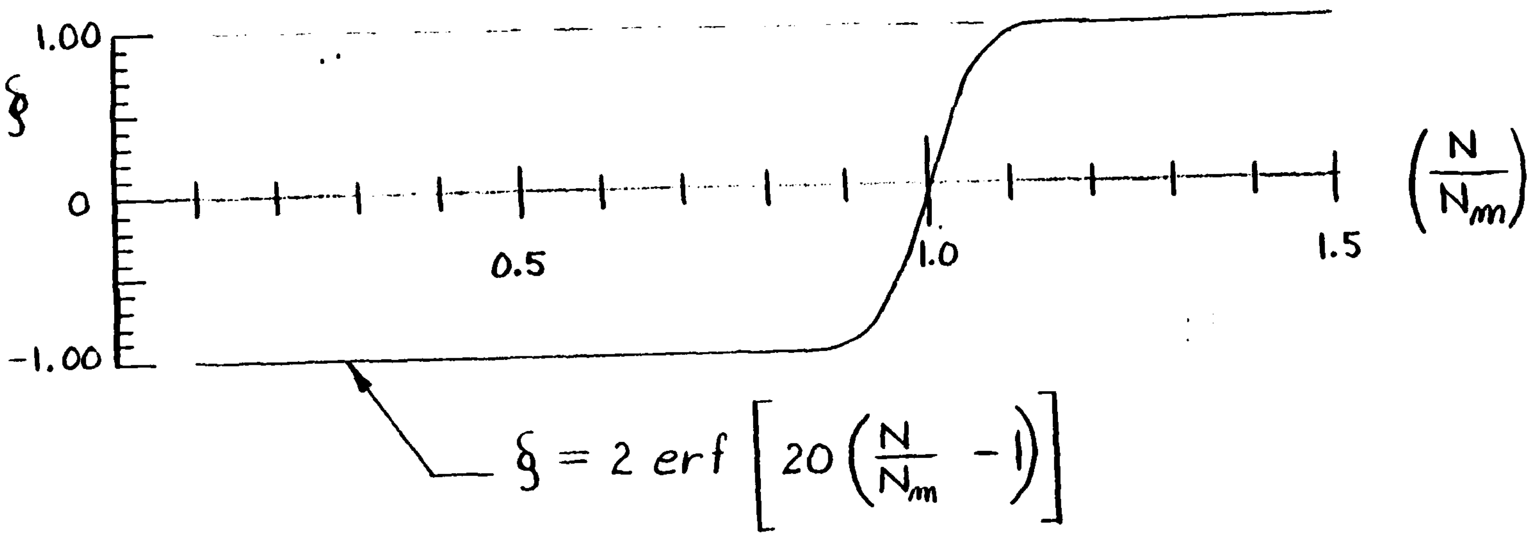
$N_{1_{MIN}}$ = minimum cycles to first failure

$$N_{1_{MIN}} = 0.5\left[\frac{\bar{\varepsilon}_\mu - 4.52\ \Delta_\varepsilon}{\overline{}_{\varepsilon} + 4.52\ \delta_\varepsilon}\right]^{\beta} \text{cycles} \tag{18}$$

σ_{N_1} = standard deviation of N_1

$$\sigma_{N_1} \cong (\bar{N}_1 - N_{1_{MIN}})/3 \tag{19}$$

FIGURE 13 CORRECTION FACTOR VERSUS N/N_m



The above equations have been included along with the previously described Monte Carlo simulation technique into one program PL-1 which is written in Basic Language.

Equivalently,

$$\delta_\sigma = \delta \quad \text{(psi)}$$
$$\Delta_\sigma = \Delta \quad \text{(psi)}$$

$$F(N) = 0.5 + \mathrm{erf}\left[\frac{2^{-1/\beta}\,\sigma'_f}{\Psi_\sigma}\left\{\left(\frac{N}{N_m}\right)^{1/\beta} - 1\right\}\right]$$

$$\Psi_\sigma = \sqrt{\Delta_\sigma^{\,2} + (2N_m)^{2/\beta}\,\delta_\sigma^{\,2} + \frac{\S\,(2N_m)^{1/\beta}\,\Delta_\sigma\,\delta_\sigma}{\sqrt{2\beta - \pi/\beta}}} \qquad \text{psi}$$

$$\overline{N}_1 = N_m\left[1 - \frac{3.7195451}{\left(\dfrac{2^{-1/\beta}\,\sigma'_f}{\Psi_\sigma}\right)}\right]^{\beta} \qquad \text{cycles}$$

$$N_{1_{MIN}} = 0.5\left[\frac{2^{-1/\beta}\,\sigma^{1}_{f} - 4.52\,\Delta_\sigma}{\overline{\sigma} + 4.52\,\delta_\sigma}\right]^{\beta} \qquad \text{cycles}$$

PL-1

SIMULATION COMPUTER PROGRAM LISTING

```
10 REM N1=AVG DUCTILITY(IN/IN)
20 REM D1=STD DEV DUCTILITY(IN/IN)
30 REM N2=AVG APPLIED STRAIN(IN/IN)
40 REM D2=STD DEV APPLIED STRAIN(IN/IN)
50 REM
60 REM B1=BETA(SLOPE PARAMETER)
70 REM W=BIN WIDTH (CYCLES"
80 REM S=SAMPLE SIZE
90 REM N8=GENERATED CYCLES AT FAILURE
100 REM      (RANDOM VARIABLE)
110 REM
120 REM X4=TALLIED AVG DUCTILITY
130 REM D5=TALLIED STD DEV DUCTILITY
140 REM X5=TALLIED APPLIED STRAIN
150 REM D6=TALLIED STD DEV APPLIED STRAIN
160 REM
170 REM F1=FAILURE PROBABILITY USING TALLIED PARAMETERS
180 N1=.135
190 D1=.000925
200 D2=.00022
210 N2=.004387
220 PRINT
230 PRINT "DESIRED PARAMETERS:"
240 PRINT "DUCTILITY:AVG,STD DEV:"
250 PRINT N1,D1
260 PRINT "APPLIED STRAIN:AVG,STD DEV:"
270 PRINT N2,D2
280 PRINT
290 L=1E7
300 N3=0
310 S8=0
320 T8=0
330 S9=0
340 T9=0
350 S1=0
360 K=1
370 B1=9.6
380 B2=1/(B1)
390 B3=2*B2
400 W=5E5
410 S=10000
420 A1=0.2548296
430 A2=-0.2844967
440 A3=1.421414
450 A4=-1.45315
460 A5=1.06141
470 DIM A(2000),B(2000),F(2000)
480 DIM S(2000),Q(2000)
```

```
490 FOR I=1 TO 2000
500 A(I)=0
510 B(I)=0
520 NEXT I
530 FOR B=1 TO S
540 U1=RND(-1)
550 U2=RND(-1)
560 Z1=SQR(-2*D1**2*LOG(U2))
570 X3=Z1*COS(6.283185*U1)+N1
580 IF X3<=0 GO TO 540
590 S8=S8+X3
600 T8=T8+X3**2
610 U3=RND(-1)
620 U4=RND(-1)
630 Z2=SQR(-2*D2**2*LOG(U4))
640 Y3=Z2*COS(6.283185*U3)+N2
650 IF Y3<=0 GO TO 610
660 S9=S9+Y3
670 T9=T9+Y3**2
680 N8=(1/2)*((X3/Y3)**B1)
690 IF N8>L THEN 710
700 L=N8
710 J=INT(((N8-N3)/W)+1)
720 IF J<=K GO TO 740
730 K=J
740 A(J)=A(J)+1
750 NEXT B
760 K6=N3-W
770 FOR I=1 TO K
780 K6=K6+W
790 B(I)=A(I)/S
800 S1=S1+A(I)
810 S(I)=S1/S
820 NEXT I
830 PRINT
840 PRINT
850 PRINT
860 X4=S8/S
870 D5=SQR((T8/S)-(X4**2))
880 X5=S9/S
890 D6=SQR((T9/S)-(X5**2))
900 N7=(1/2)*((X4/X5)**B1)
910 B5=(2*N7)**B3
920 PRINT "TALLIED PARAMETERS:"
930 PRINT "DUCTILITY:AVG,STD DEV:"
940 PRINT X4,D5
950 PRINT "APPLIED STRAIN:AVG,STD DEV:"
960 PRINT X5,D6
970 PRINT
```

```
980 PRINT "CYCLES AT FIRST FAILURE=";INT(L+.5)
990 PRINT
1000 PRINT
1010 PRINT "N(MEDIAN)=";INT(N7+.5)
1020 PRINT
1030 PRINT "SAMPLE SIZE=";S
1040 PRINT
1050 PRINT "BETA=";B1
1060 PRINT
1070 D8=SQR(2*B1-(3.14159/B1))
1080 D9=((2*N7)**B2)/D8
1090 P0=(D5**2)+(B5*D6**2)
1100 P2=SQR((D1**2)+(D2**2))
1110 N9=(1/2)*((N1/N2)**B1)
1120 FOR I=1 TO K
1130 N=(I-1)*W
1140 P1=SQR(P0+SGN((N/N7)-1)*D9*D5*D6)
1150 M3=(X4/P1)*((((N+W)/N7)+B2)-1)
1160 M5=M3
1170 IF M3>=0 GO TO 1190
1180 M5=-M3
1190 T2=1/(1+0.2316418*M5)
1200 C5=(A2*T2**2)+(A3*T2**3)+(A4*T2**4)
1210 C6=(A1*T2)+C5+(A5*T2**5)
1220 A8=1-(C6*(EXP(-(M5**2)/2)))
1230 E1=(1/2)*ABS(A8)
1240 E2=E1
1250 IF M3>=0 GO TO 1270
1260 E2=-E1
1270 M4=(N1/P2)*((((N+W)/N9)+B2)-1)
1280 M6=M4
1290 IF M4>=0 GO TO 1310
1300 M6=-M4
1310 T3=1/(1+0.2316418*M6)
1320 C7=(A2*T3**2)+(A3*T3**3)+(A4*T3**4)
1330 C8=(A1*T3)+C7+(A5*T3**5)
1340 A9=1-(C8*(EXP(-(M6**2)/2)))
1350 E3=(1/2)*ABS(A9)
1360 E4=E3
1370 IF M4>=0 GO TO 1390
1380 E4=-E3
1390 F(I)=.5+E2
1400 F2=0.5+E4
1410 NEXT I
1420 Q(1)=F(1)*S
1430 FOR I=2 TO K
1440 Q(I)=(F(I)-F(I-1))*S
1450 NEXT I
1460 PRINT " ","TALLIED","CALC'D","TALLIED","CALC'D"
1470 PRINT "CYCLES","FAILURES","FAILURES","F(N)","F(N)"
1480 FOR I=1 TO K
1490 N=(I-1)*W
1500 IF S(I)<.995 THEN 1520
1510 IF A(I)<.5 THEN 1530
1520 PRINT N,A(I),INT(Q(I)+.5),S(I),INT(1E4*F(I)+.5)/1E4
1530 NEXT I
1540 END
```

PLASTIC REGION HISTOGRAM RESULTS

Table II shows the desired and tallied parameters for eleven cases in the low cycle fatigue region. $\beta \approx 2$ for most structural materials. The sample size of N_f for each case is 10,000 to minimize the variances of the results. N_m was chosen to cover the upper and lower ends of the low cycle fatigue regions. Cases $\boxed{1}$, $\boxed{3}$, $\boxed{5}$ and $\boxed{7}$ have $\delta_\varepsilon = 0$. The theoretical results for these cases were rigorously derived. The variances Δ_ε and δ_ε were chosen in some cases to be large enough to cause the cycles to first failure to be significantly lower than N_m. See figures 14 - 23.

The curves of figures 22 and 23 have the same parameters. The theoretical expressions for Ψ_ε are different however. There is a better fit (especially in the region of first failures between the tallied and theoretical results of figure 23 where the proposed form equation (15) for Ψ_ε is used.

It can be seen that there is good agreement between the theoretical results using the proposed fatigue life equations and the tallied Monte Carlo simulation results.

TABLE II

DESIRED VERSUS TALLIED PARAMETERS:
LOW CYCLE FATIGUE

CASE	$\overline{\varepsilon}_\mu$ DESIRED / TALLIED	Δ_ϵ DESIRED / TALLIED	$\overline{\varepsilon}$ DESIRED / TALLIED	δ_ϵ DESIRED / TALLIED	N_m DESIRED / TALLIED
1	0.2	0.01	0.0024	0	3472
	0.19976	0.00996	0.0023999	1.29 E-5	3464
2	0.2	0.01	0.0024	0.0003	3472
	0.19982	0.01014	0.002398	0.0002978	3471
3	0.2	0.03	0.0024	0	3472
	0.19986	0.0298	0.0023444	1.29 E-5	3467
4	0.2	0.03	0.0024	0.0003	3472
	0.19948	0.03029	0.002348	0.000298	3460
5	0.2	0.01	0.00632	0	500
	0.20008	0.00999	0.00632	4.48 E-5	501
6	0.2	0.01	0.00632	0.00079	500
	0.20005	0.010003	0.006325	0.000795	500
7	0.2	0.03	0.00632	0	500
	0.199751	0.03004	0.00632	4.48 E-5	499
8	0.2	0.03	0.00632	0.00079	500
	0.20003	0.03009	0.006324	0.000795	500
9	0.2	0.02	0.0024	0.00048	3472
	0.19966	0.0201	0.002397	0.000485	3469
10	0.2	0.02	0.0024	0.00048	3472
	0.19964	0.0203	0.002395	0.000478	3474
11	0.2	0.04	0.0024	0.00048	3472
	0.14468	0.0393	0.002399	0.000478	3464

$\overline{\varepsilon}_\mu$, $\overline{\varepsilon}$, Δ_ϵ δ_ϵ ∿ in/in

N_m ∿ CYCLES

SAMPLE SIZE: 10,000

FIGURE 14 HISTOGRAM OF N_f : CASE $\boxed{1}$

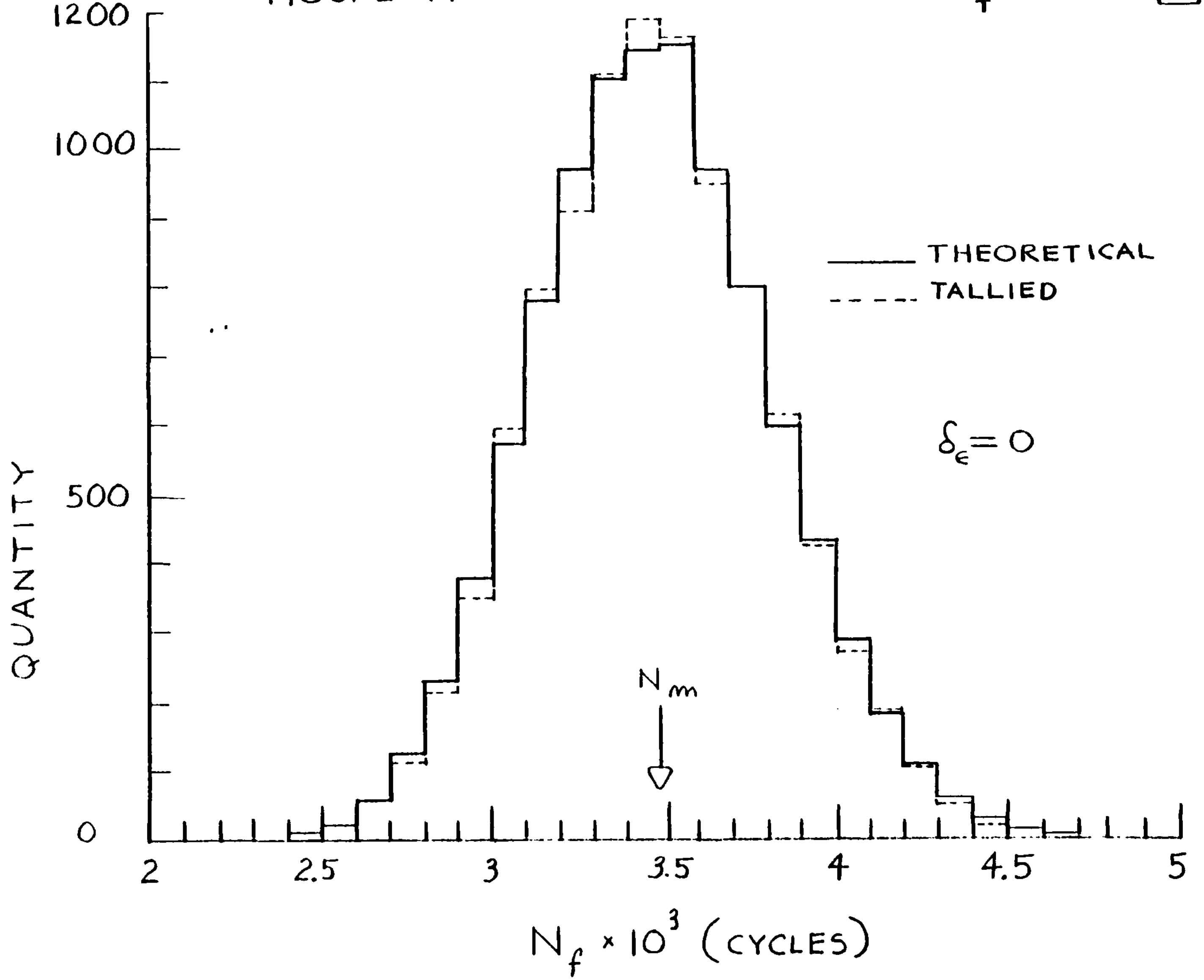


FIGURE 15

HISTOGRAM OF N_f : CASE $\boxed{2}$

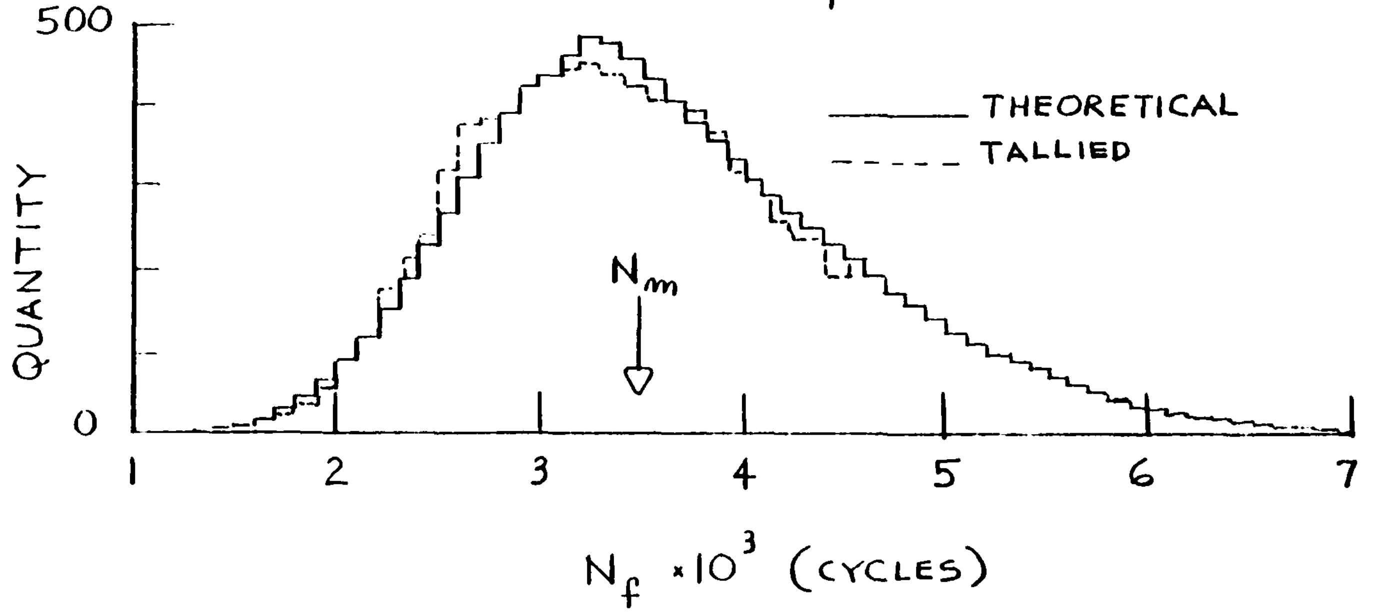


FIGURE 16
HISTOGRAM OF N_f : CASE $\boxed{3}$

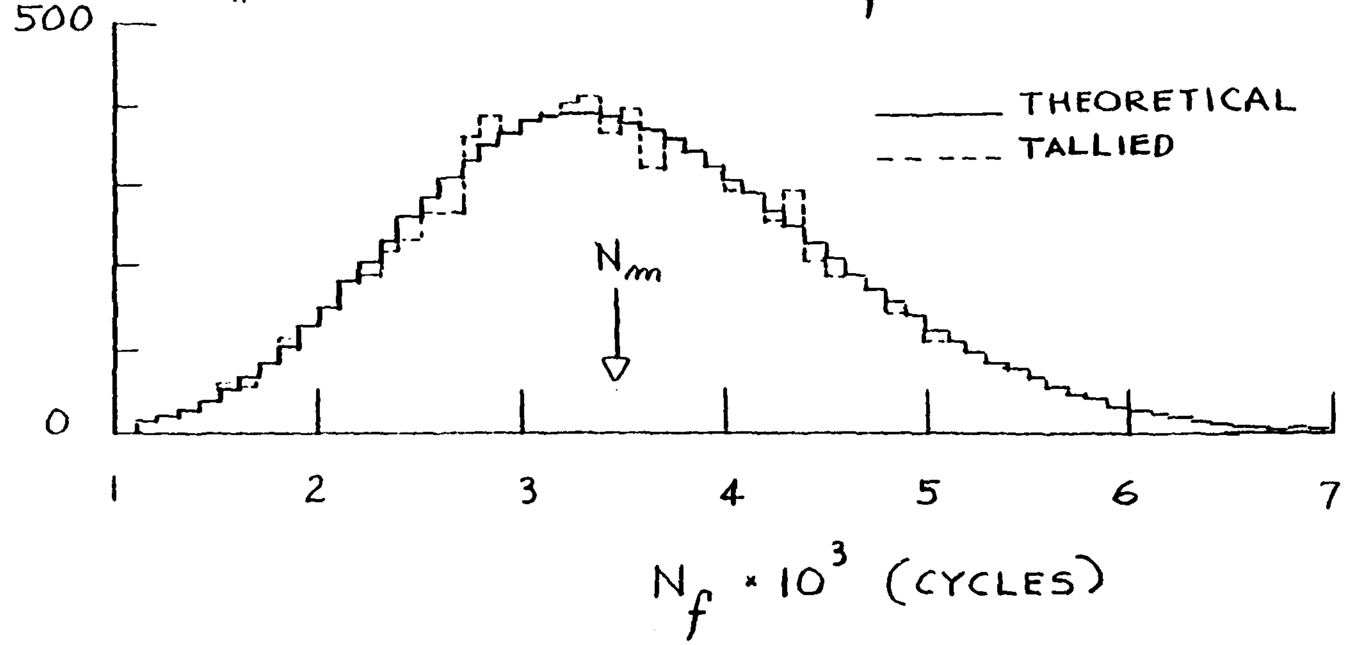


FIGURE 17

HISTOGRAM OF N_f : CASE $\boxed{4}$

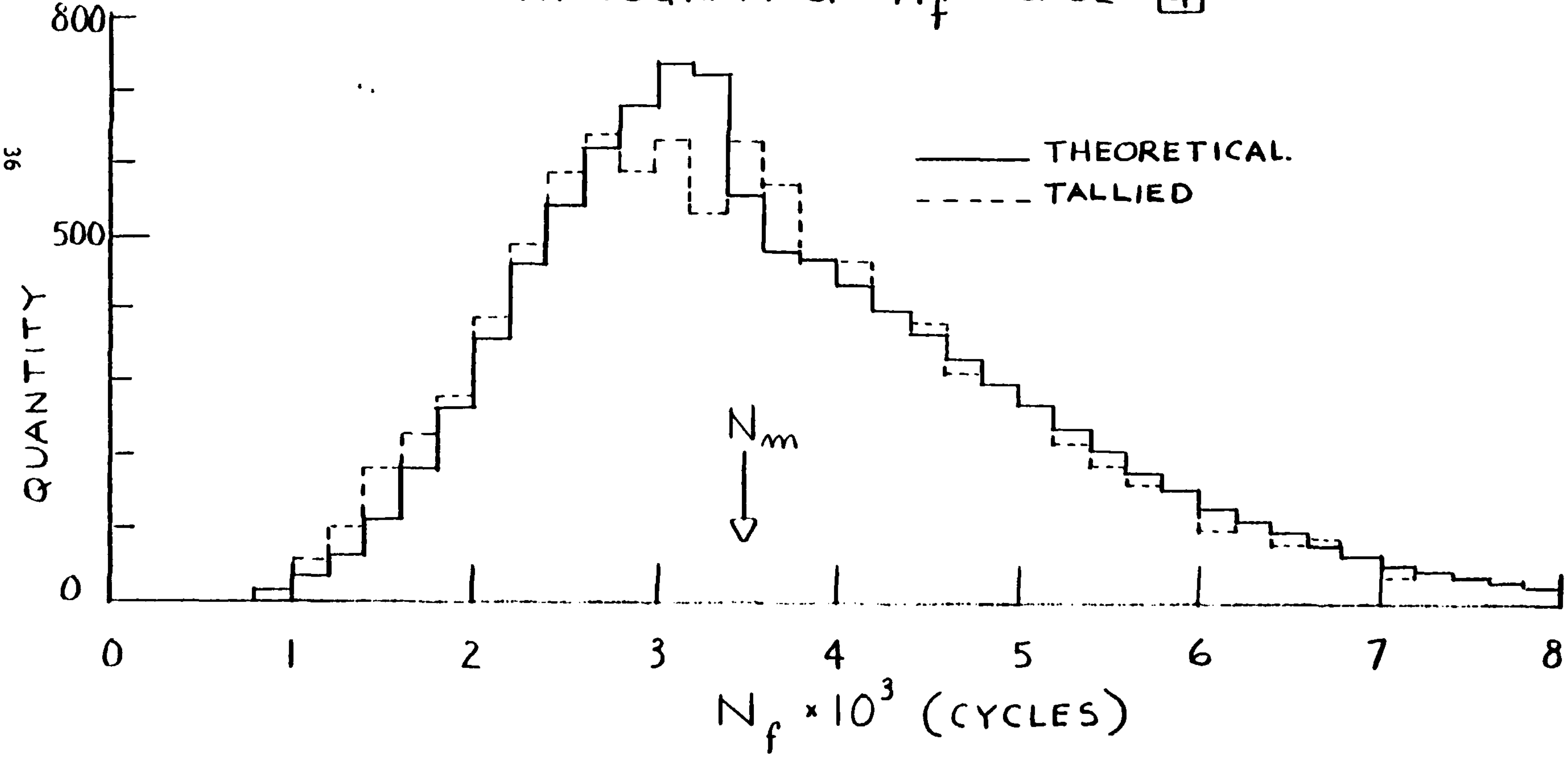


FIGURE 18 HISTOGRAM OF N_f : CASE $\boxed{5}$

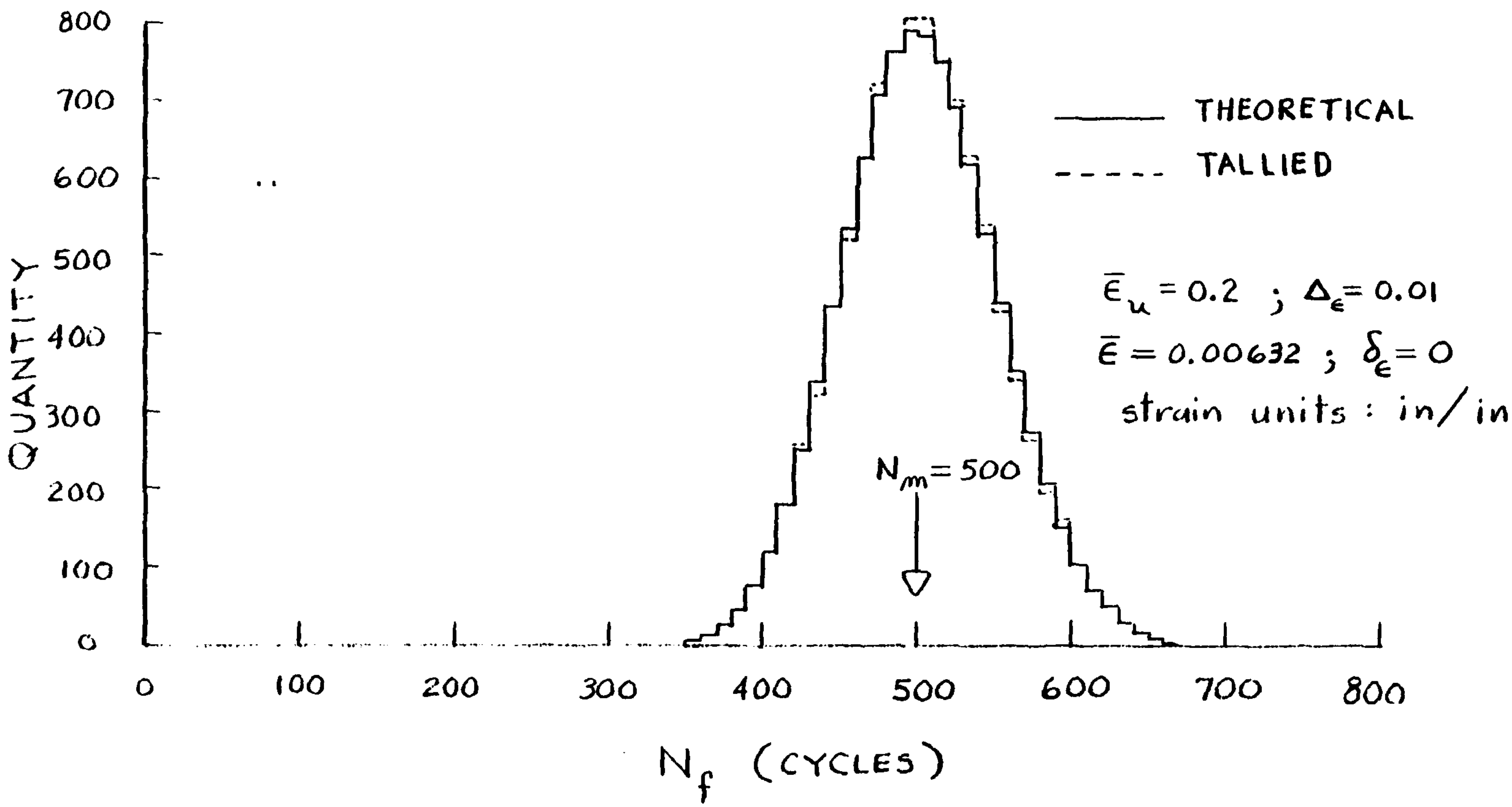


FIGURE 19 HISTOGRAM OF N_f : CASE $\boxed{6}$

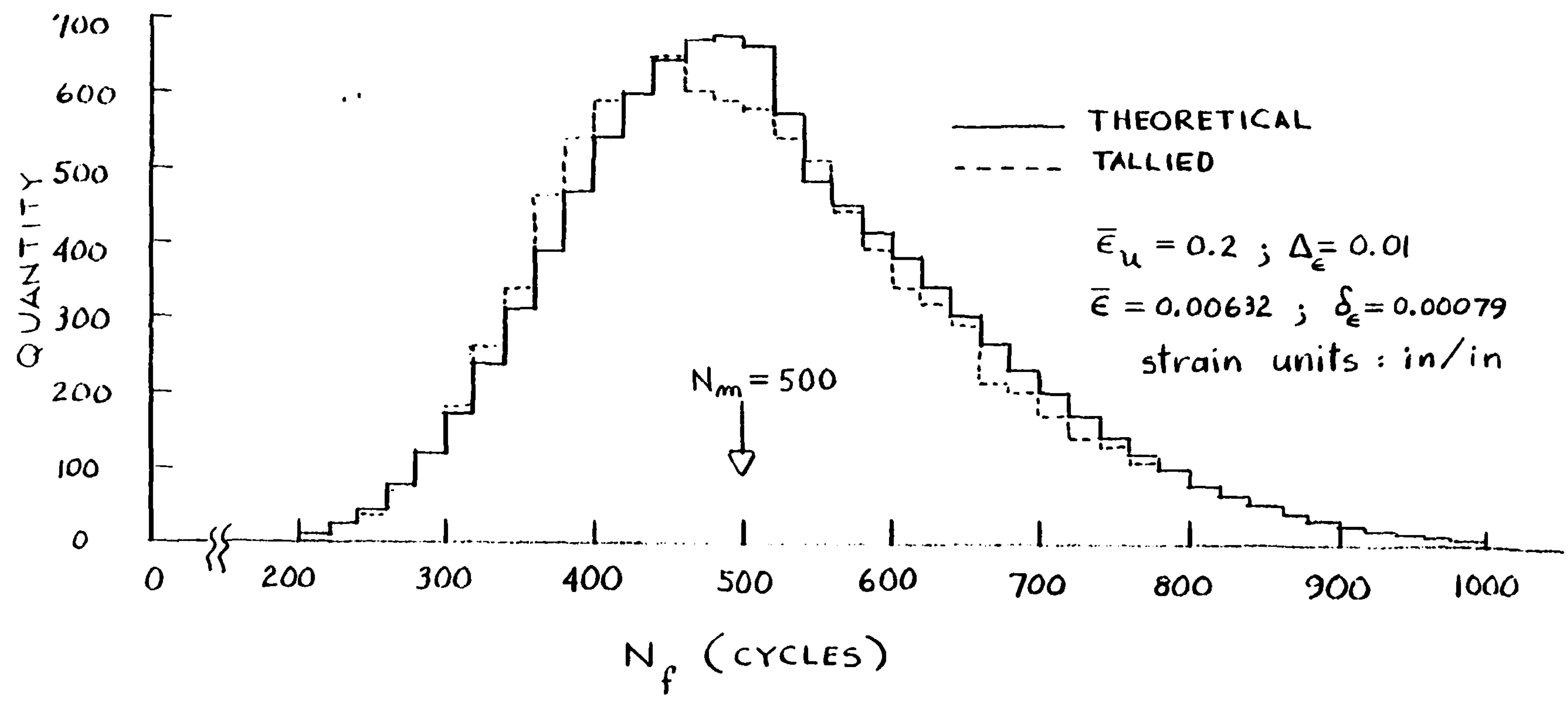


FIGURE 20
HISTOGRAM OF N_f : CASE $\boxed{7}$

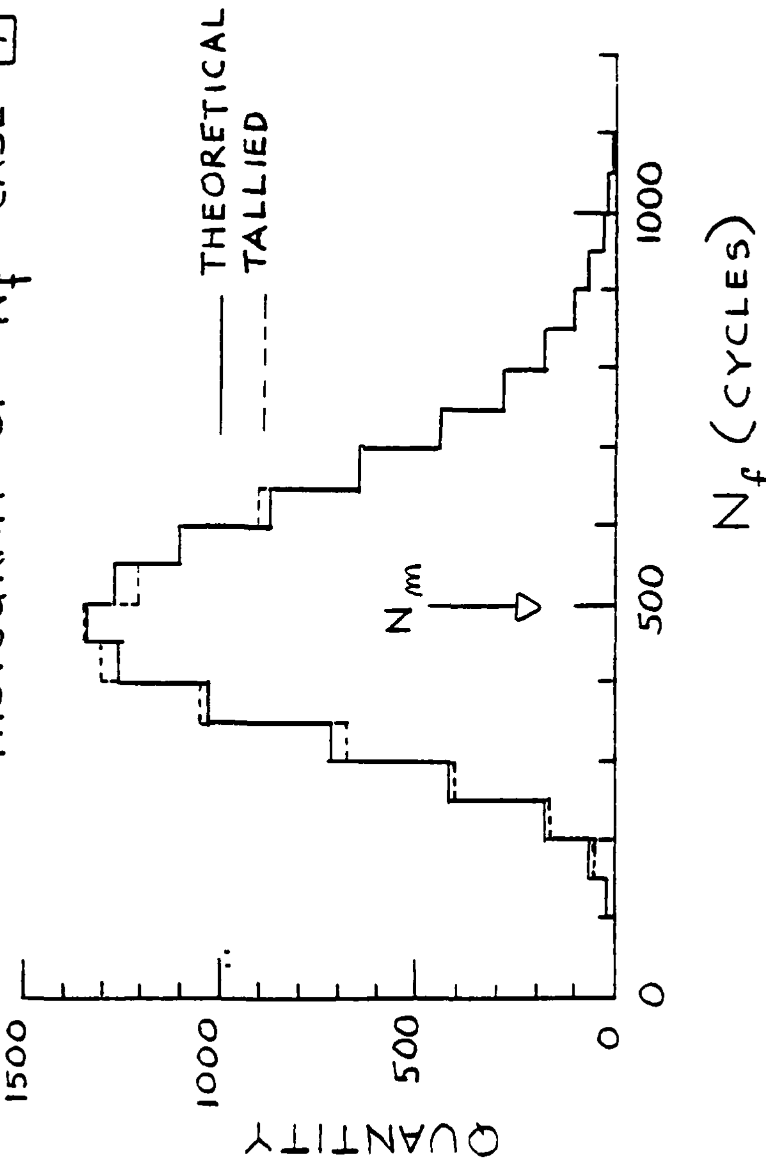


FIGURE 21

HISTOGRAM OF N_f : CASE $\boxed{8}$

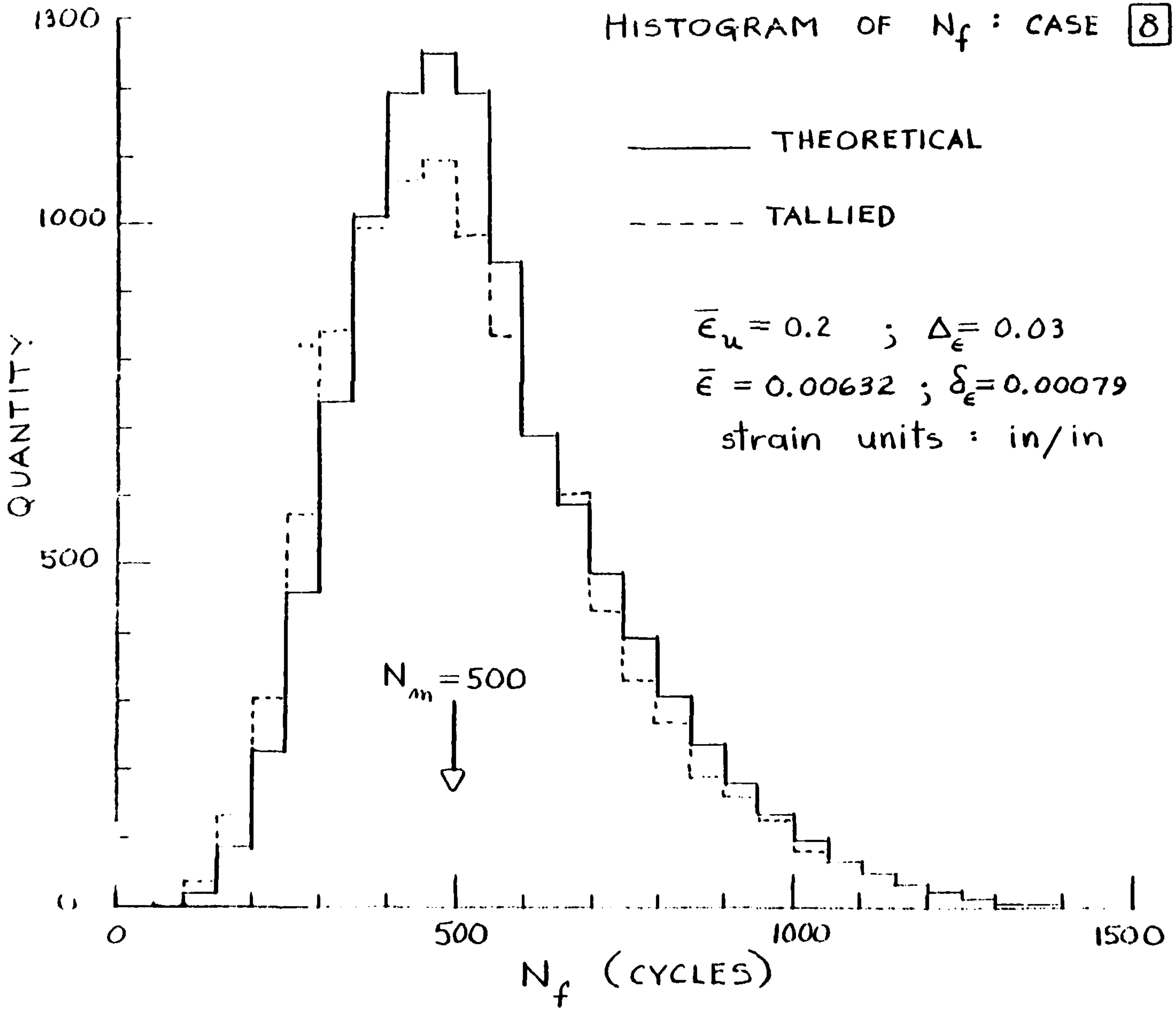


FIGURE 22 HISTOGRAM OF N_f : CASE $\boxed{9}$

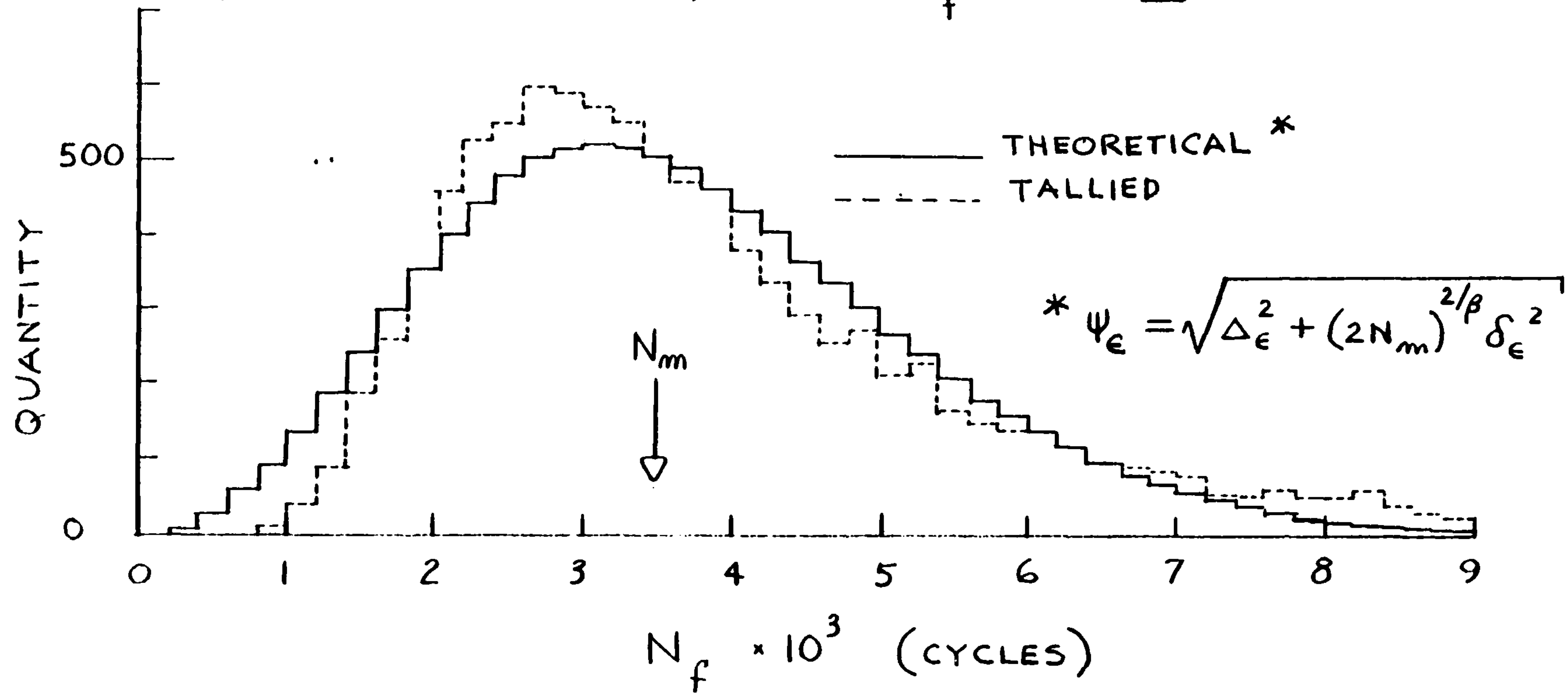


FIGURE 23 HISTOGRAM OF N_f : CASE $\boxed{10}$

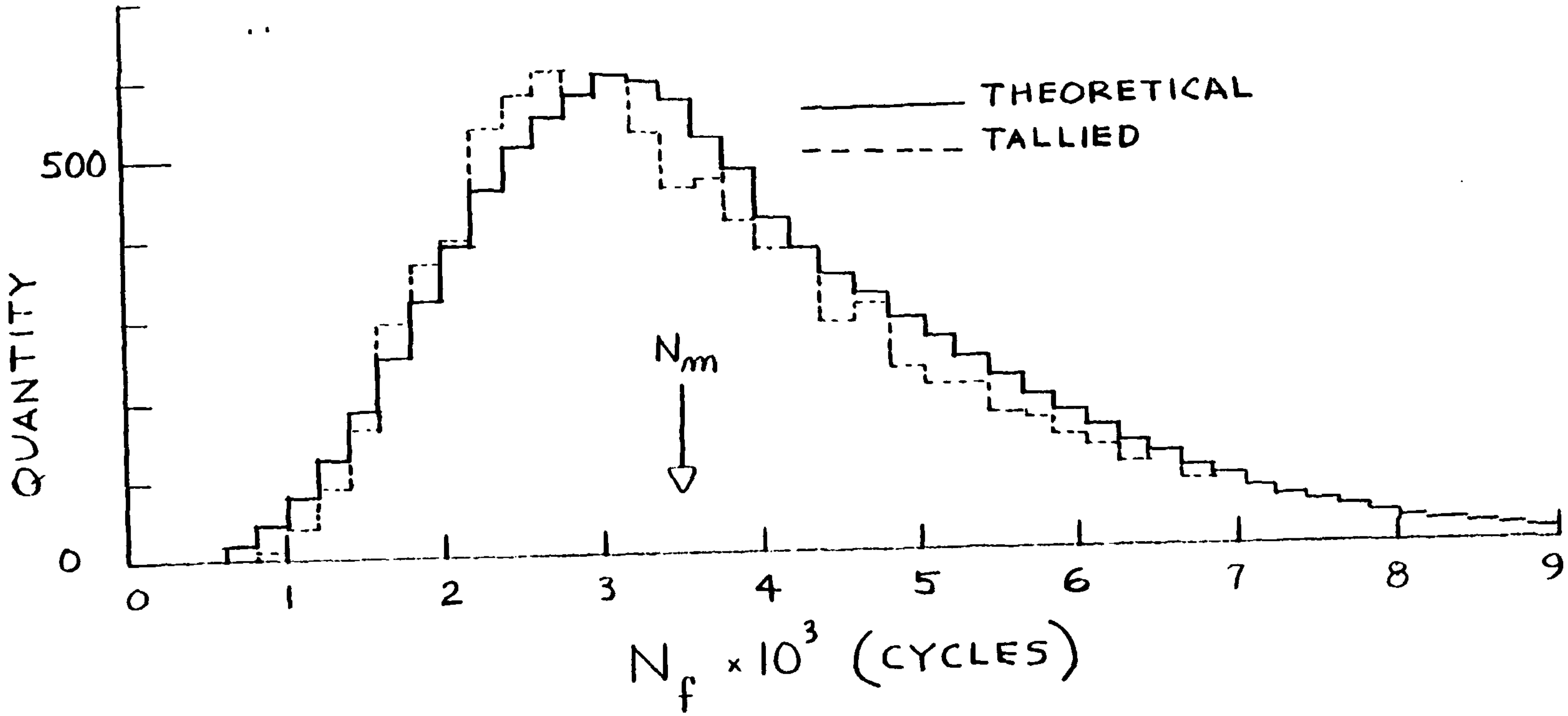
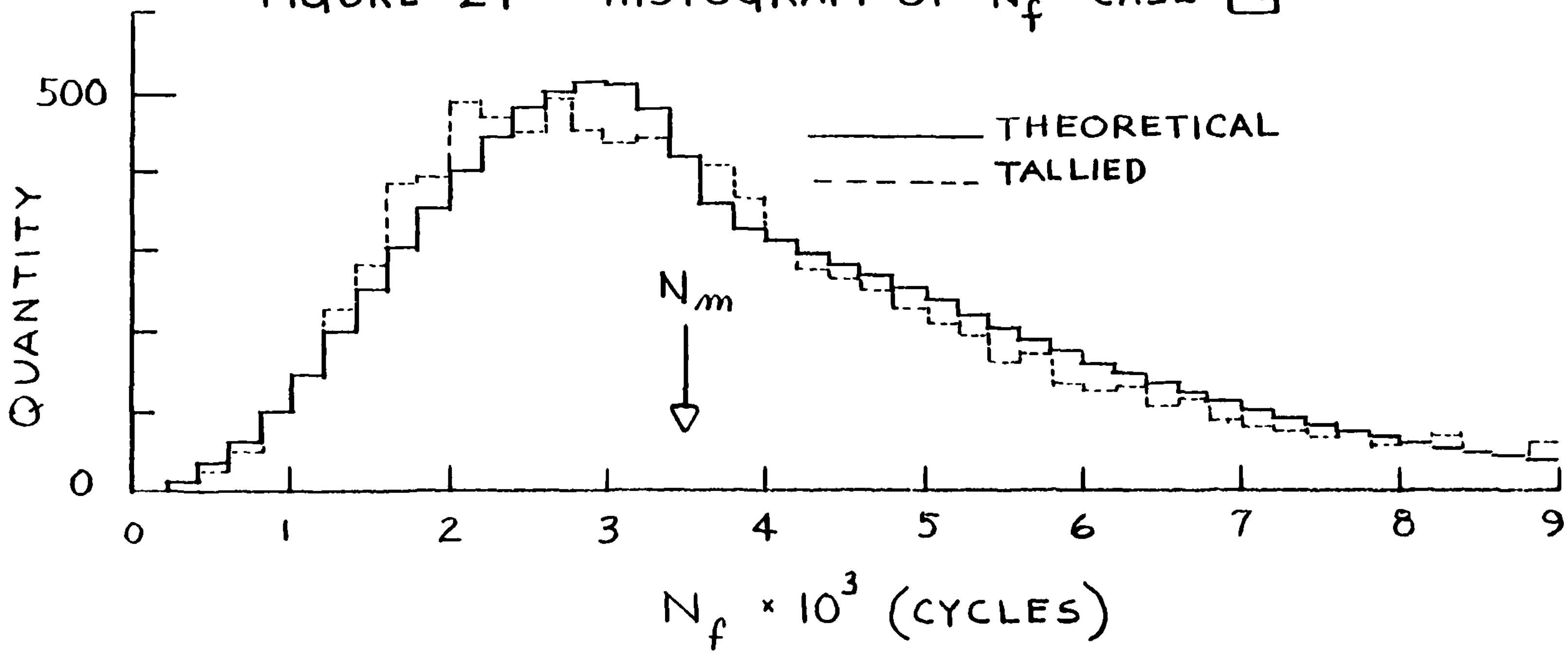


FIGURE 24 HISTOGRAM OF N_f : CASE $\boxed{11}$



ELASTIC REGION HISTOGRAM RESULTS

Table III shows the desired and tallied parameters for eight cases in the high cycle fatigue region. Three values of β are used. $\beta = 9.6$ represents 7075-T6 Aluminum Alloy. $\beta = 12.1$ represents G-10 Epoxy Fiberglass. $\beta = 22.37$ represent AZ31B Magnesium Alloy. Such a range covers ductile to brittle materials. N_m is chosen to cover from the lower to upper end of the high cycle region. The standard deviations are large enough to cause the cycles to first failure to be much less than N_m.

Figures 25 through 32 show the theoretical results using the proposed fatigue life equations and the tallied simulation results. Figure 31 is the one exception.

$\Psi'_\varepsilon = \sqrt{\Delta_\varepsilon^2 + \delta_\varepsilon^2}$ is used for case 7, not the proposed equation (15). Figure 31 shows that the theoretical histogram is reasonably accurate but not nearly as accurate as those using equation (15) for Ψ_ε. In the vicinity of the early failures the theoretical curve is non-conservative. This is the reason that Ψ'_ε was discarded.

In all other cases the proposed results are accurate compared to the tallied results.

Figures 29 and 32 show the effect of large β on the spread of cycles to failure. Figure 32 in particular shows a preponderance of failures in the early life region, much less than N_m.

TABLE III

DESIRED VERSUS TALLIED PARAMETERS:
HIGH CYCLE FATIGUE

CASE	$\bar{\varepsilon}_\mu$ DESIRED / TALLIED	Δ_ϵ DESIRED / TALLIED	$\bar{\varepsilon}$ DESIRED / TALLIED	δ_ϵ DESIRED / TALLIED	N_m DESIRED / TALLIED	β
1	0.0185	0.000925	0.004387	0	500,000	
	0.0185	0.000919	0.0043869	4.8 E-6	500,976	9.6
2	0.0185	0.000925	0.004387	0.00022	500,000	
	0.01851	0.000944	0.004389	0.000223	501,073	9.6
3	0.0185	0.000925	0.0034515	0	5,000,000	
	0.01851	0.000947	0.0034514	1.9 E-5	5,036,934	9.6
4	0.0185	0.000925	0.0034515	0.0001726	5,000,000	
	0.0185	0.000919	0.0034518	0.0001729	5,000,445	9.6
5	0.0185	0.000925	0.004387	0.00022	18,201,500	
	0.01848	0.000932	0.004385	0.000223	19,150,800	12.1
6	0.0185	0	0.0034515	0.0001726	5,000,000	
	0.0184995	0.000129	0.0034526	0.0001733	4,982,646	9.6
7	0.0185	0.000925	0.004387	0.00022	500,000	
	0.01851	0.000936	0.004388	0.000224	502,200	9.6
8	0.0021582	0.0001079	0.0011638	0.0005819	500,000	
	0.0021574	0.0001086	0.0011634	0.00005886	499,930	22.37

$\bar{\varepsilon}_\mu$, $\bar{\varepsilon}$, Δ_ϵ, δ_ϵ ∿ in/in

N_m ∿ CYCLES

SAMPLE SIZE: 10,000

FIGURE 25
HISTOGRAM OF N_f : CASE (1)

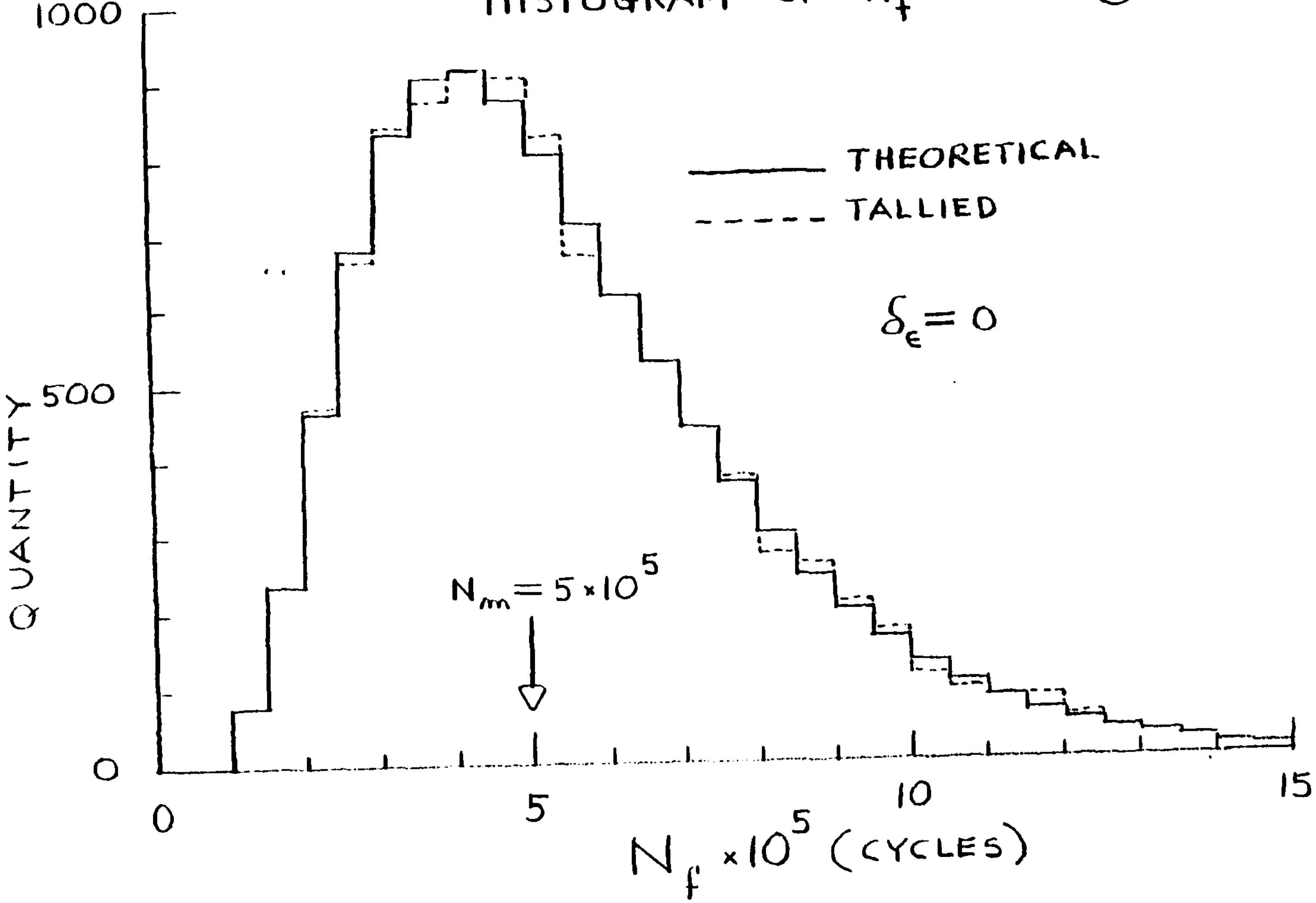


FIGURE 26

HISTOGRAM OF N_f : CASE 2

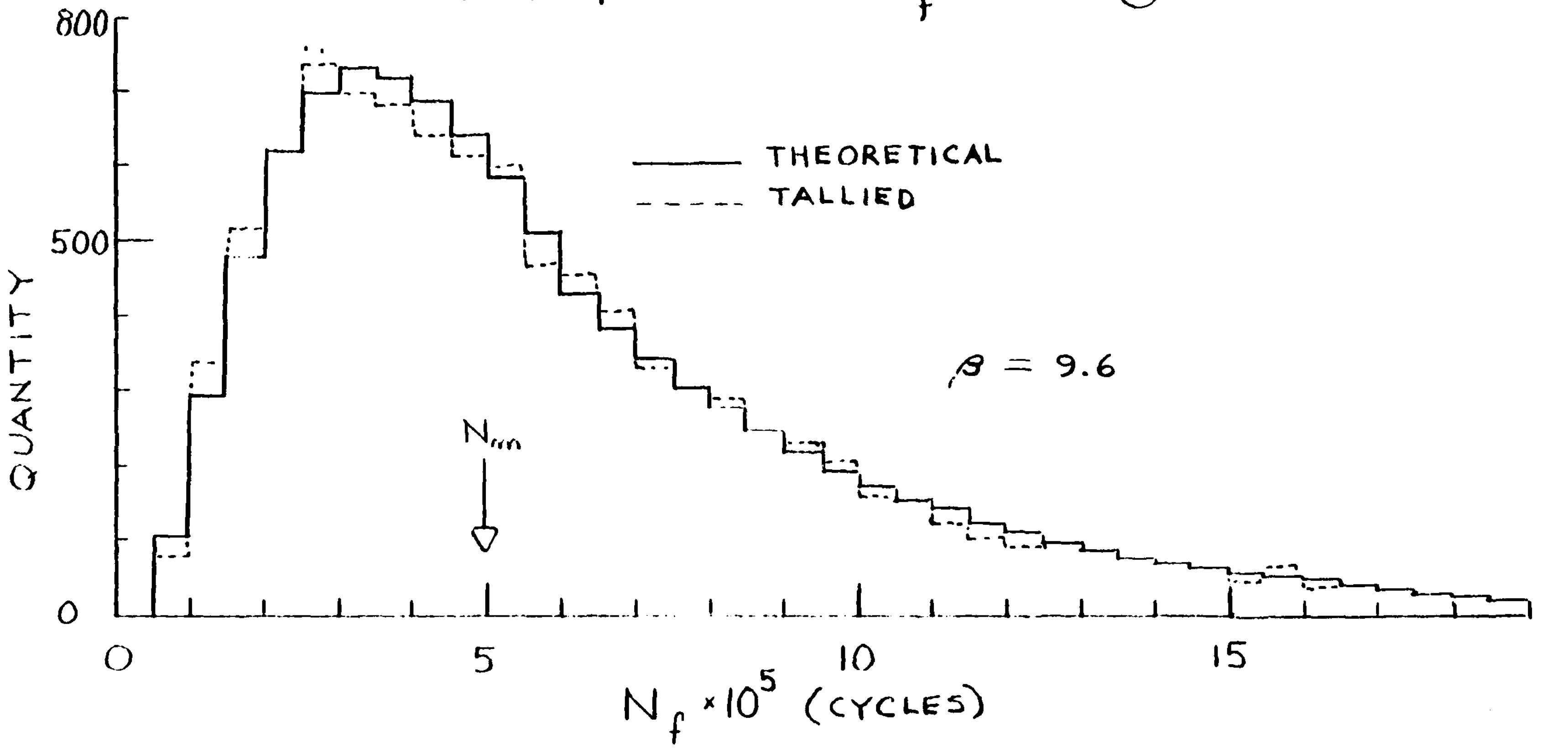


FIGURE 27
HISTOGRAM OF N_f : CASE 3

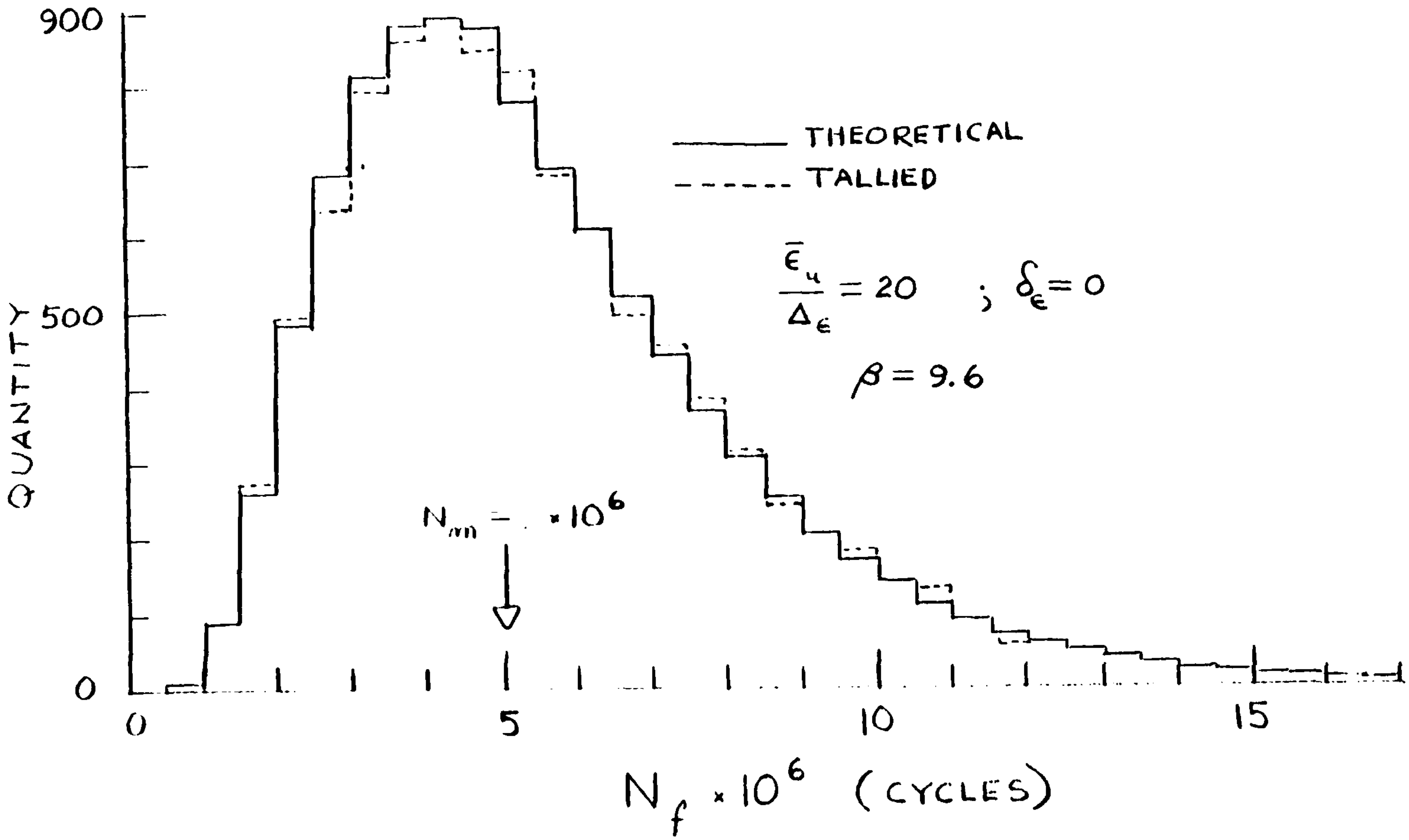


FIGURE 28

HISTOGRAM OF N_f : CASE 4

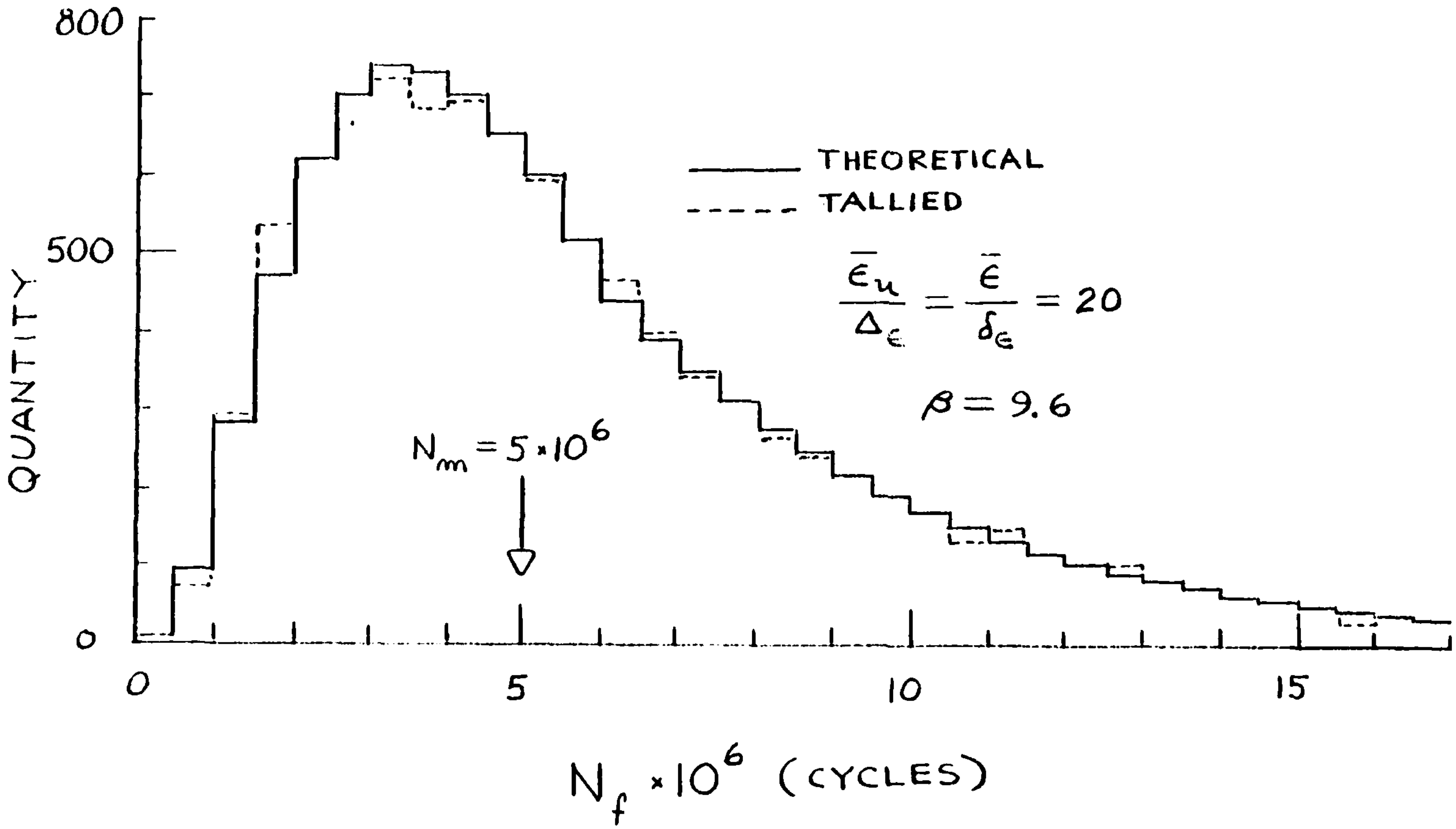


FIGURE 29

HISTOGRAM OF N_f : CASE 5

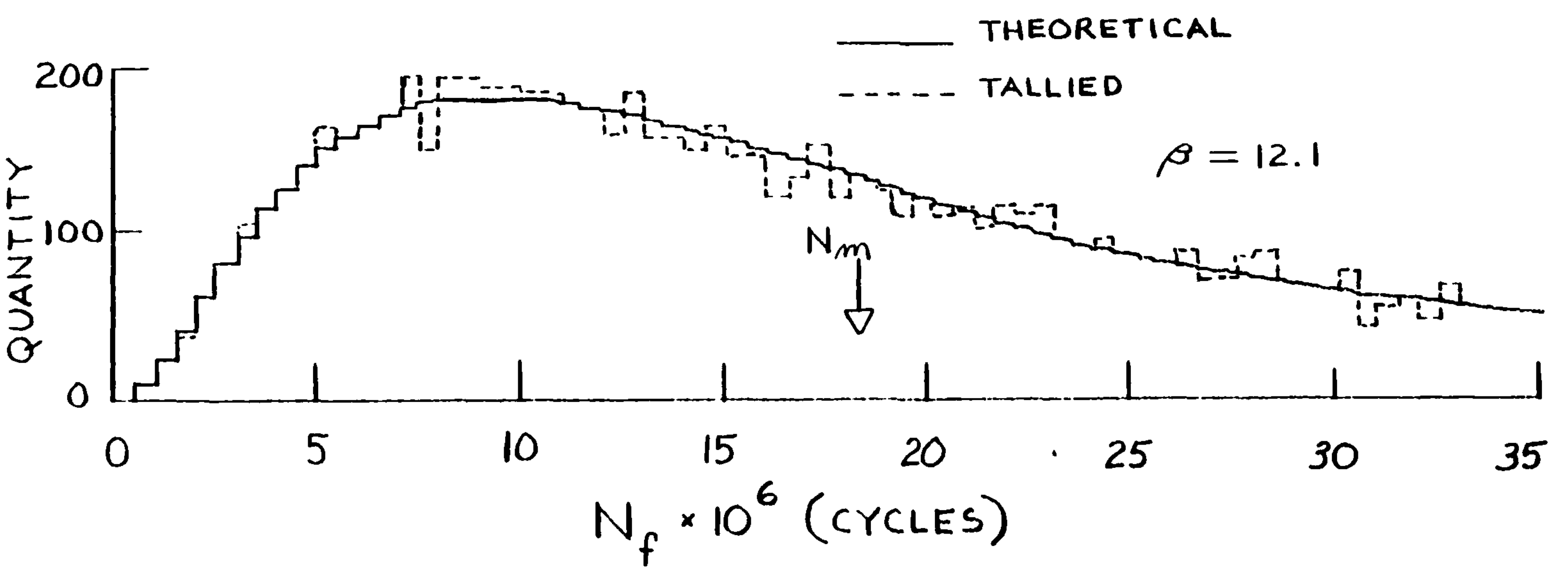


FIGURE 30
HISTOGRAM OF N_f : CASE 6

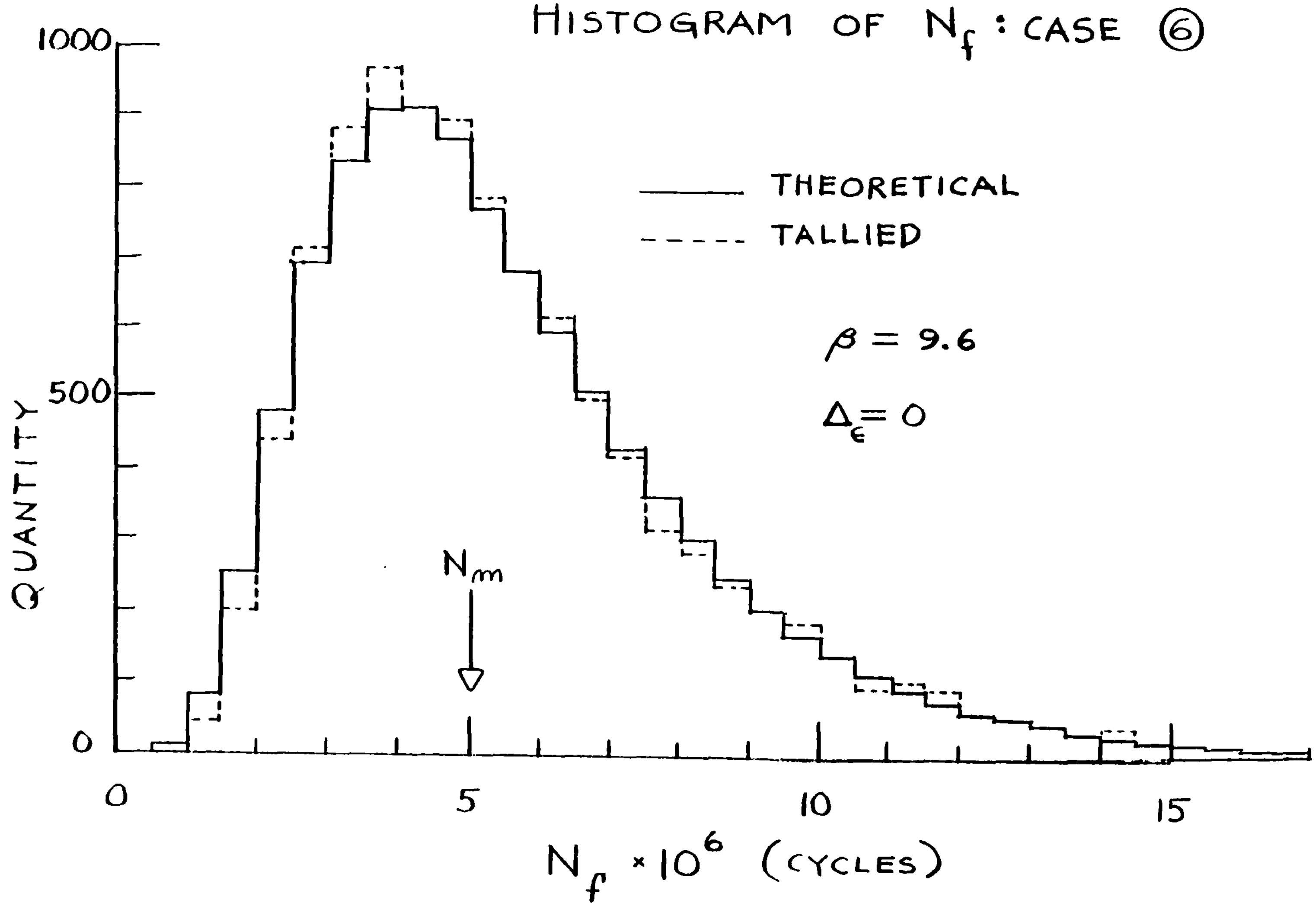


FIGURE 31
HISTOGRAM OF N_f : CASE 7

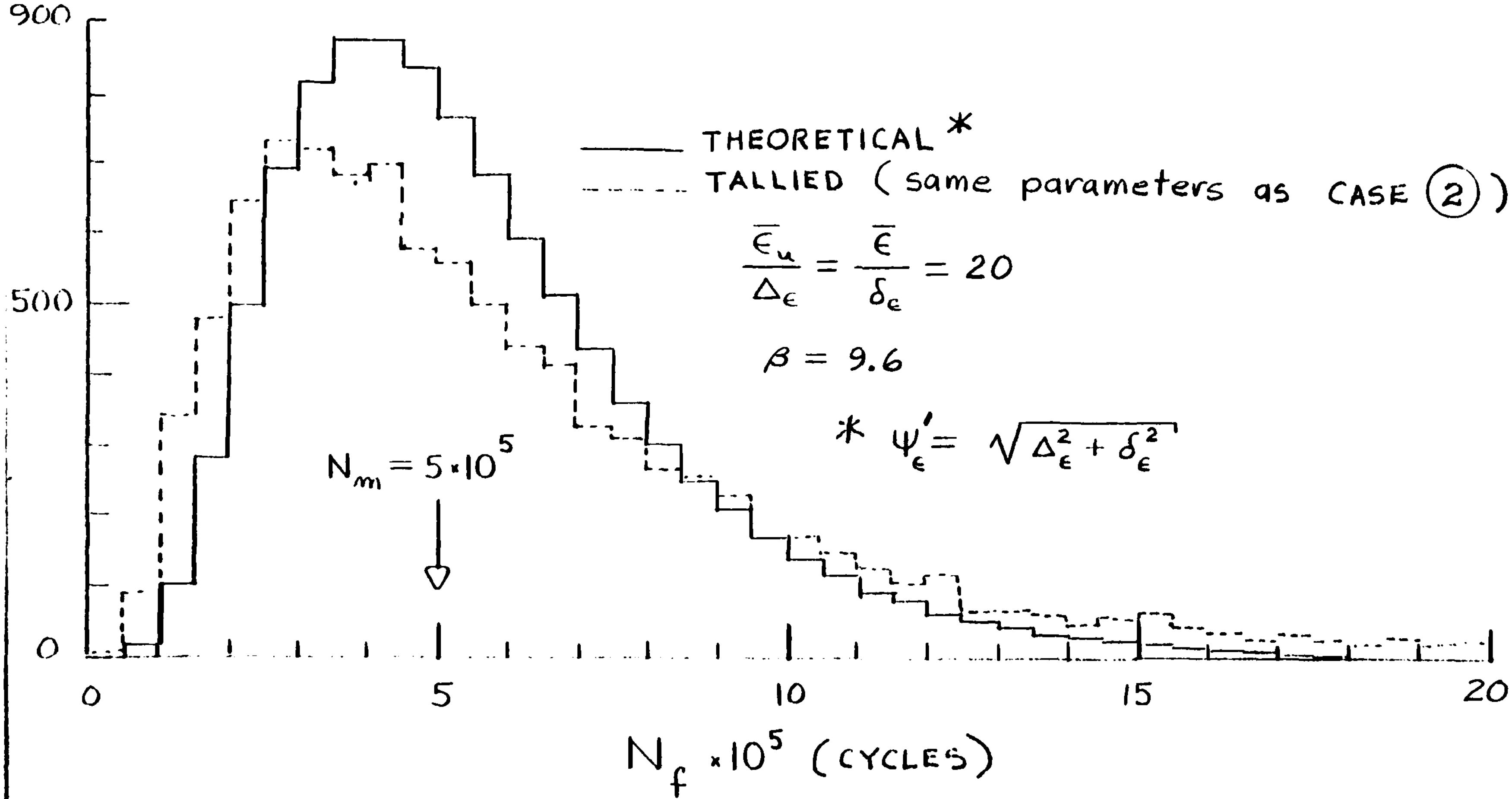
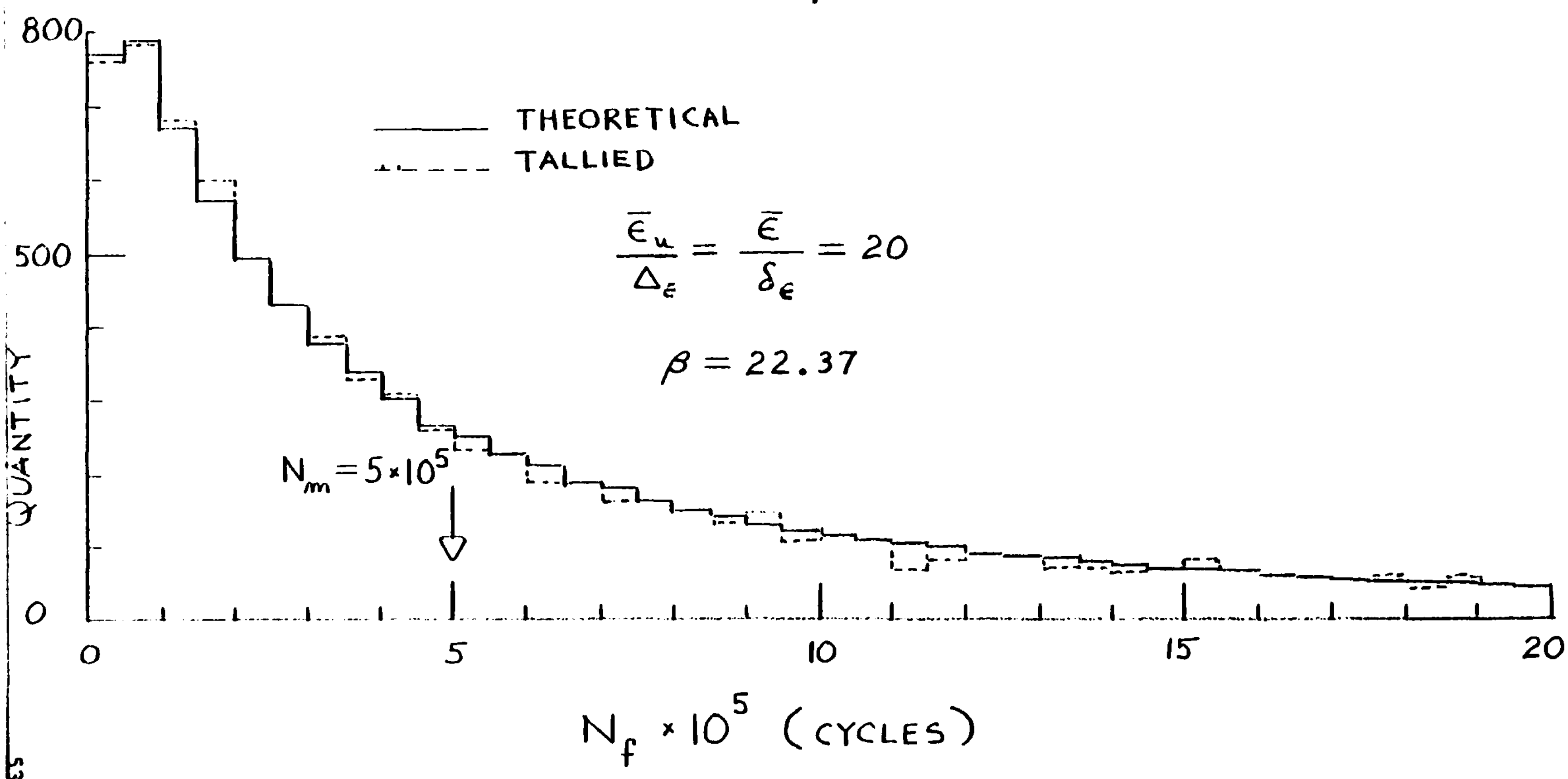


FIGURE 32
HISTOGRAM OF N_f : CASE 8



CYCLES TO FIRST FAILURE RESULTS

Appendix B shows the derivation of the expressions for $\overline{N}_1$, $N_{1_{MIN}}$ and σ_{N_1}. PL-2 is a listing of the Monte Carlo simulation program to tally the cycles to first failure. This program generates N_f samples in the same way as PL-1. The program first generates 10,000 N_f samples and selects the lowest valued sample as the first sample of the cycles to first failure N_1. This process is repeated 19 more times. A total of 20 x 10,000 = 200,000 N_f samples are generated to obtain 20 samples of N_1. $\overline{N}_1$ and σ_{N_1} are measured for these 20 samples. The smallest single value of N_1 is called $N_{1_{MIN}}$. Equations (17), (18) and (19) are used to calculate the expected corresponding values. Table IV compares tallied and calculated results. Most of the results show excellent agreement between tallied and calculated values. Some comparisons are good. Two are poor. Such a range in quality of agreement is considered to be caused by the low sample size for N_1 of 20 and not by an inherent inaccuracy of the proposed equations. To significantly increase the sample size of N_1 would be prohibitive in terms of computer time and cost. The overall good agreement already shown does not warrant any further effort.

PL-2 was modified by changing lines 130 and 140 from 10,000 and 20 to 400 and 500 respectively. Thus 400 x 500 = 200,000 N_f samples were generated to acquire 500 samples of N_1. This was done to obtain the shape of the N_1 histogram. Figures 33 and 34 show two such histogram shapes. The shapes look surprisingly like those of N_f only backwards (i.e. rotated about the median value of N).

Equations (17), (18) and (19) are considered to be accurate.

PL-2 CYCLES TO FIRST FAILURE
PROGRAM LISTING

```
10 N1=.0185
20 D1=.000925
30 D2=.00022
40 N2=.004387
50 PRINT "DESIRED PARAMETERS:"
60 PRINT "DUCTILITY:AVG,STD DEV:"
70 PRINT N1,D1
80 PRINT "APPLIED STRAIN:AVG,STD DEV:"
90 PRINT N2,D2
100 PRINT
110 B1=9.6
120 B2=1/B1
130 S=10000
140 K=20
150 S1=0
160 S2=0
170 FOR I=1 TO K
180 L=1E8
190 FOR B=1 TO S
200 U1=RND(-1)
210 U2=RND(-1)
220 Z1=SQR(-2*D1**2*(LOG(U2)))
230 X3=Z1*COS(6.283185*U1)+N1
240 IF X3<=0 THEN 200
250 U3=RND(-1)
260 U4=RND(-1)
270 Z2=SQR(-2*D2**2*(LOG(U4)))
280 Y3=Z2*COS(6.283185*U3)+N2
290 IF Y3<=0 THEN 250
300 N8=(.5)*((X3/Y3)**B1)
310 IF N8>L THEN 330
320 L=N8
330 NEXT B
340 L5=INT(L+.5)
350 PRINT L5;
360 S1=S1+L
370 S2=S2+L↑2
380 NEXT I
390 A1=S1/K
400 V1=SQR(S2/K-A1↑2)
410 A2=INT(A1+.5)
420 V2=INT(V1+.5)
430 PRINT
440 PRINT "SAMPLE SIZE=";K
450 PRINT
460 PRINT "N1(AVG)=";A2
470 PRINT "N1(STD DEV)=";V2
480 END
```

TABLE IV COMPARISON OF CYCLES TO FIRST FAILURE RESULTS

Case	$\bar{N}_1$ TALL'D	$\bar{N}_1$ CALC'D	$N_{1\ MIN}$ TALL'D	$N_{1\ MIN}$ CALC'D	σ_{N_1} TALL'D	σ_{N_1} CALC'D
$\boxed{1}$	2255	2300	2026	2080	73	73
$\boxed{2}$	1398	1090	1211	849	88	80
$\boxed{3}$	644	678	275	360	145	106
$\boxed{4}$	532	568	316	147	99	140
$\boxed{5}$	328	331	304	300	11	10
$\boxed{6}$	199	157	164	122	13	12
$\boxed{7}$	92	98	48	52	19	15
$\boxed{8}$	71	82	54	21	8	20
$\boxed{9}$	777	576	649	288	71	96
$\boxed{10}$	777	576	649	288	71	96
$\boxed{11}$	158	69	13	9	60	20
1	61,916	69,356	39,116	42,749	8,235	8,869
2	35,177	32,507	12,374	6,014	8,807	8,831
3	662,338	693,556	547,341	427,421	84,734	88,712
4	356,562	326,511	201,893	60,428	64,844	88,694
5	615,847	565,935	410,963	69,451	139,375	165,495
6	947,484	693,326	800,307	706,843	83,542	- 4,505
8	981	734	442	17	389	239

All values are in units of CYCLES

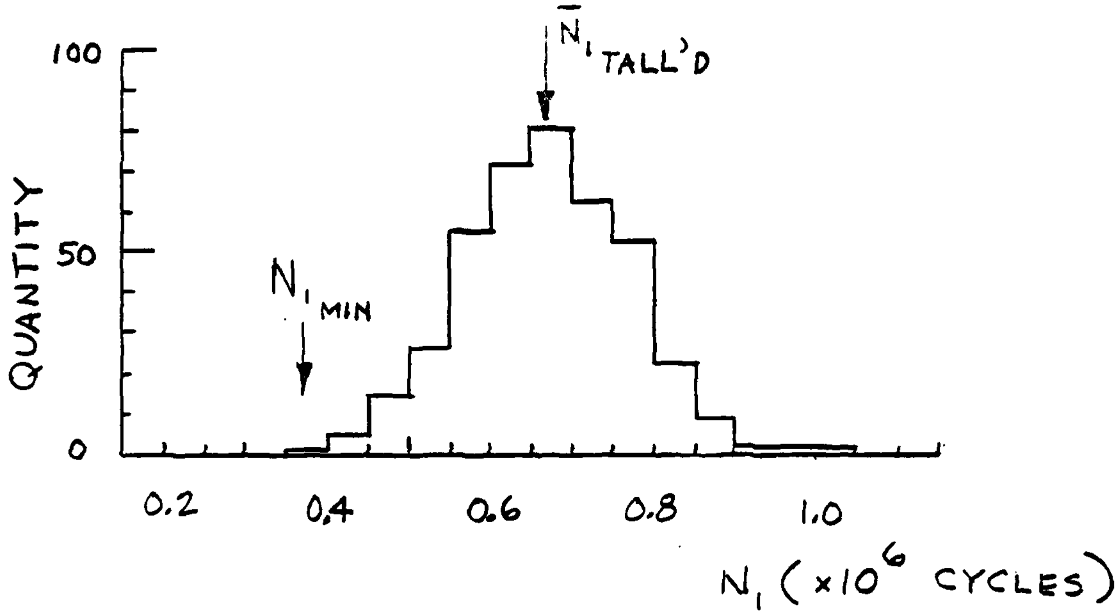


FIG. 33 HISTOGRAM OF N_1 : CASE ③

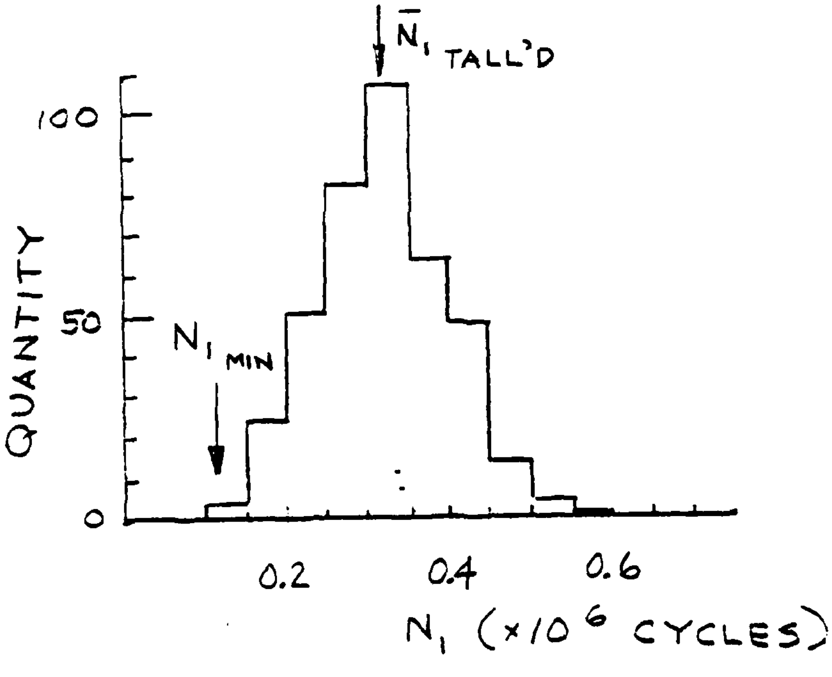


FIG. 34 HISTOGRAM OF N_1 : CASE ④

COMPARISON WITH EMPIRICAL DATA

The proposed fatigue life expressions have previously been shown to agree well with the Monte Carlo simulation tallied results. Now the theoretical and tallied results will be compared with the empirical results reported in references {2}, {8} and {10}. In reference {8}J.T. Broch describes fatigue life test results of G-10 fiberglass single-degree-of-freedom end mass cantilever beams subjected to random stresses. A sample size of 100 beams was used for the tests. The test parameters are as follows:

$\bar{\sigma} = 12.2$ ksi; $\delta = 0.348$ ksi; $\Delta = 1.75$ ksi

$\bar{C} = 33$ ksi; $\beta = 12.1$; $E = 2700$ ksi

The corresponding strain parameters are

$$\bar{\varepsilon}_{\mu} = 2^{1/\beta} \frac{\bar{C}}{E} = 0.0129427 \text{ in/in} \quad ; \quad \sigma'_f = 2^{1/\beta}\,\bar{A}$$

$$\bar{\varepsilon} = \frac{\bar{\sigma}}{E} = 0.0045 \text{ in/in}$$

$$\Delta_{\varepsilon} = \Delta/_{E} = 0.000647 \text{ in/in}$$

$$\delta_{\varepsilon} = \delta\ /_{E} = 0.000129 \text{ in/in}$$

Figure 35 shows a comparison of the theoretical and tallied histograms for the above parameters. Large variances in the tallied are noted. However, the overall shapes are in general agreement. Figure 36 compares theoretical and empirical data. Again large variances are noted in the empirical data. The overall shapes are in general agreement. Figure 37 compares the empirical and tallied histograms. They too generally are in agreement with each other. Figure 38 shows that the variance of the tallied data is smoothed out considerably as expected by increasing the sample size from 100 to 10,000. This indicates that the previous relatively large variances for the tallied and empirical data is an expected result of the small sample size of 100.

FIGURE 35
HISTOGRAM OF N_f : J.T. BROCH EXAMPLE

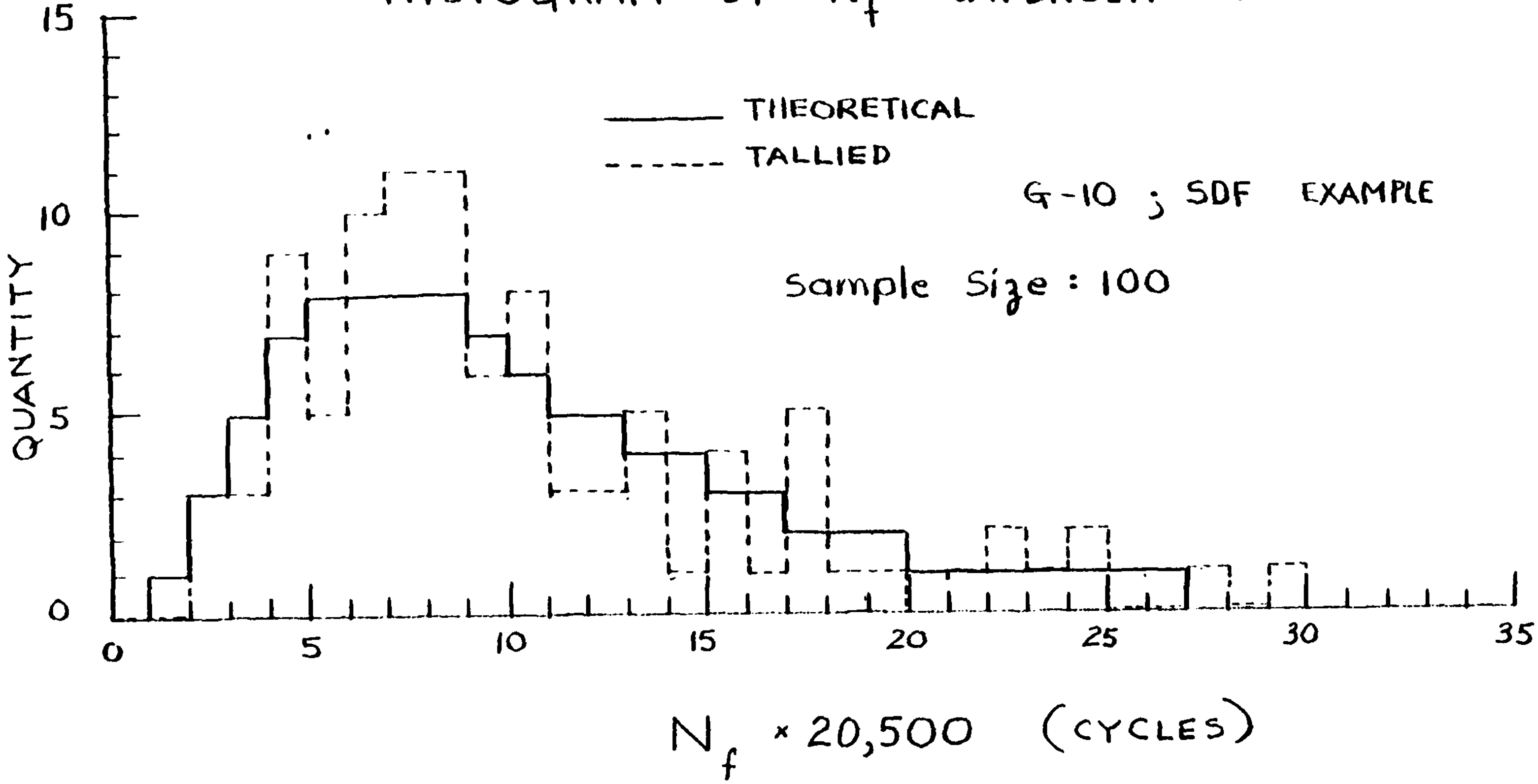


FIGURE 36
HISTOGRAM OF N_f : J.T. BROCH DATA

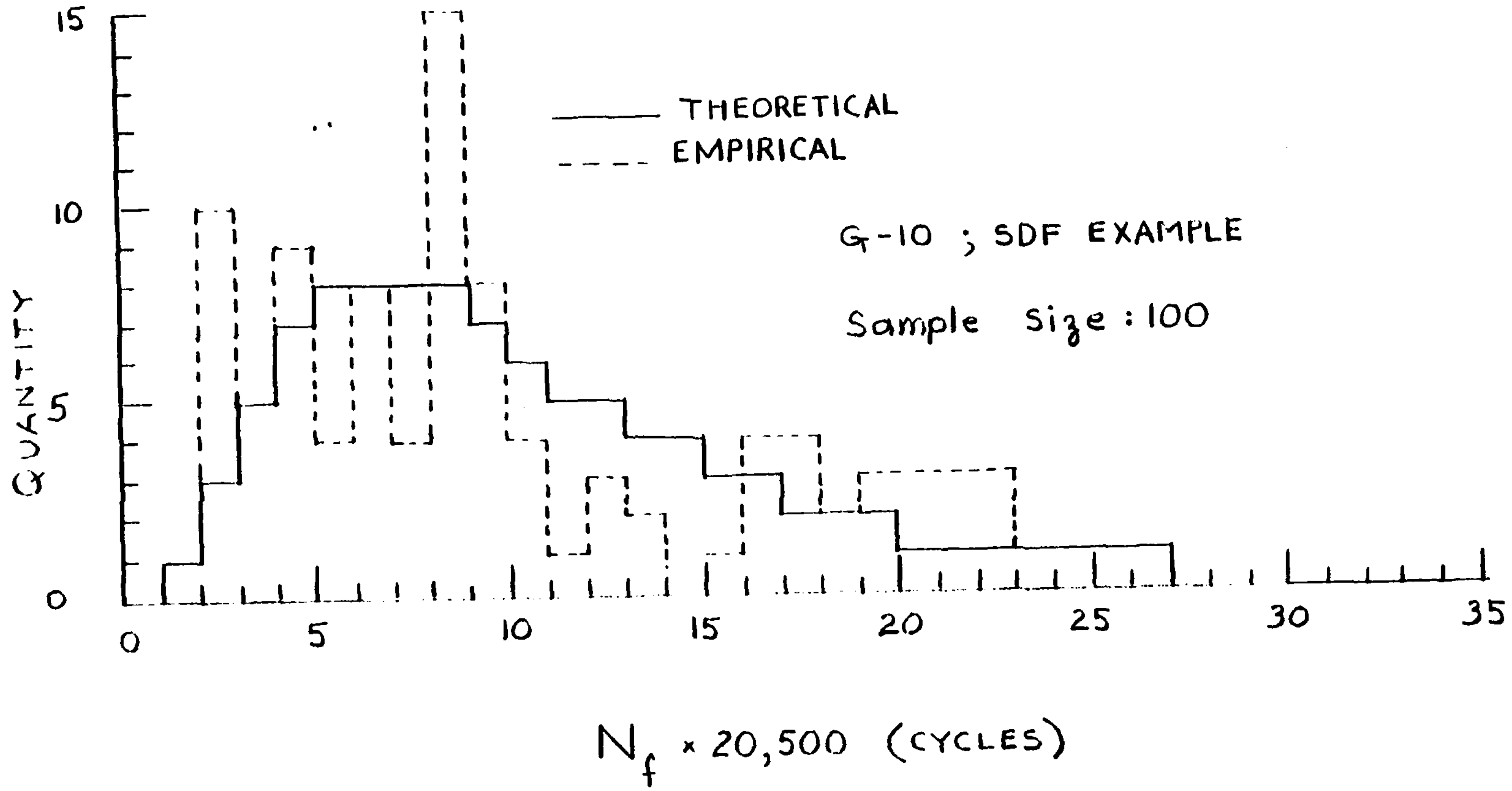


FIGURE 37

EMPIRICAL-TALLIED HISTOGRAMS :
J.T. BROCH EXAMPLE

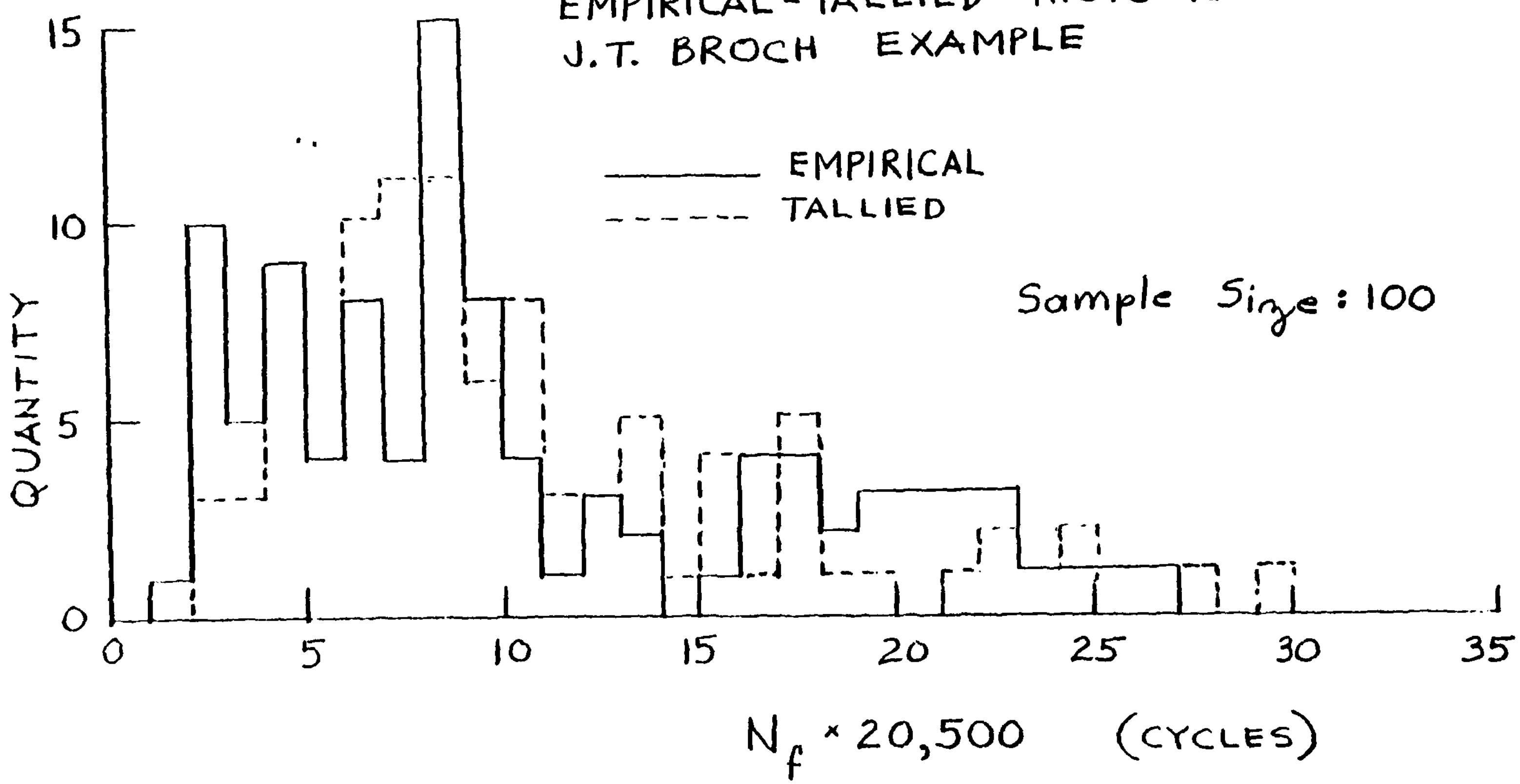


FIGURE 38
HISTOGRAM OF N_f: G-10, SDF EXAMPLE

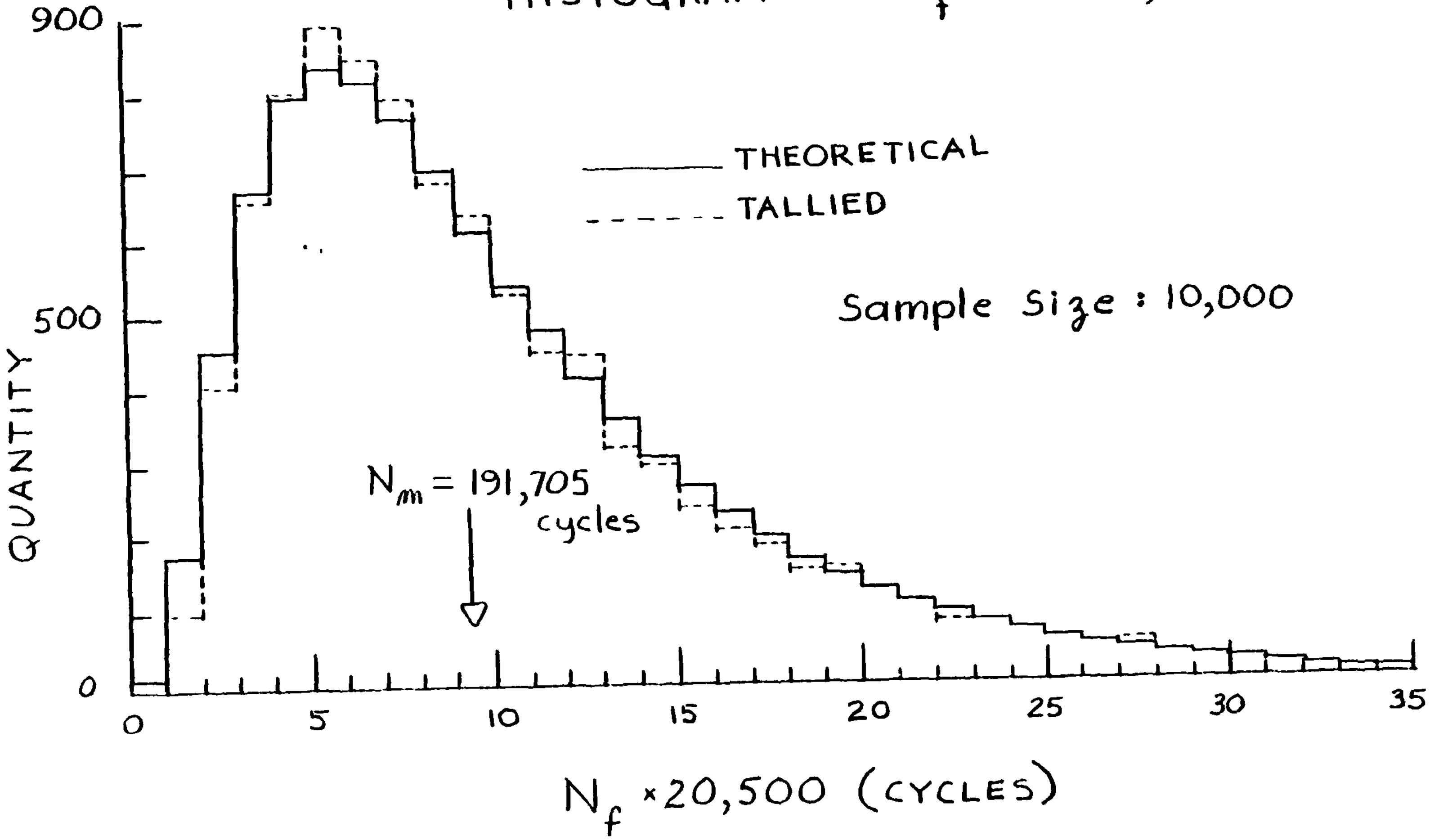


Figure 39 is a histogram of N_1. Also shown are calculated and tallied values of $\overline{N}_1$ and N_1 empirical. Quantitatively

$$\overline{N}_{1_{cal'd}} = 33,983 \text{ cycles}$$

$$\overline{N}_{1_{Tallied}} = 37,914 \text{ cycles}$$

$$\overline{N}_{1_{empirical}} = 42,950 \text{ cycles}$$

All of the data indicates good agreement between theoretical, Monte Carlo and empirical results.

Figure 40 shows additional empirical fatigue failure data {10}. Again the theoretical results are in good agreement with empirical results.

FIGURE 39

HISTOGRAM OF N_1 : J.T. BROCH EXAMPLE

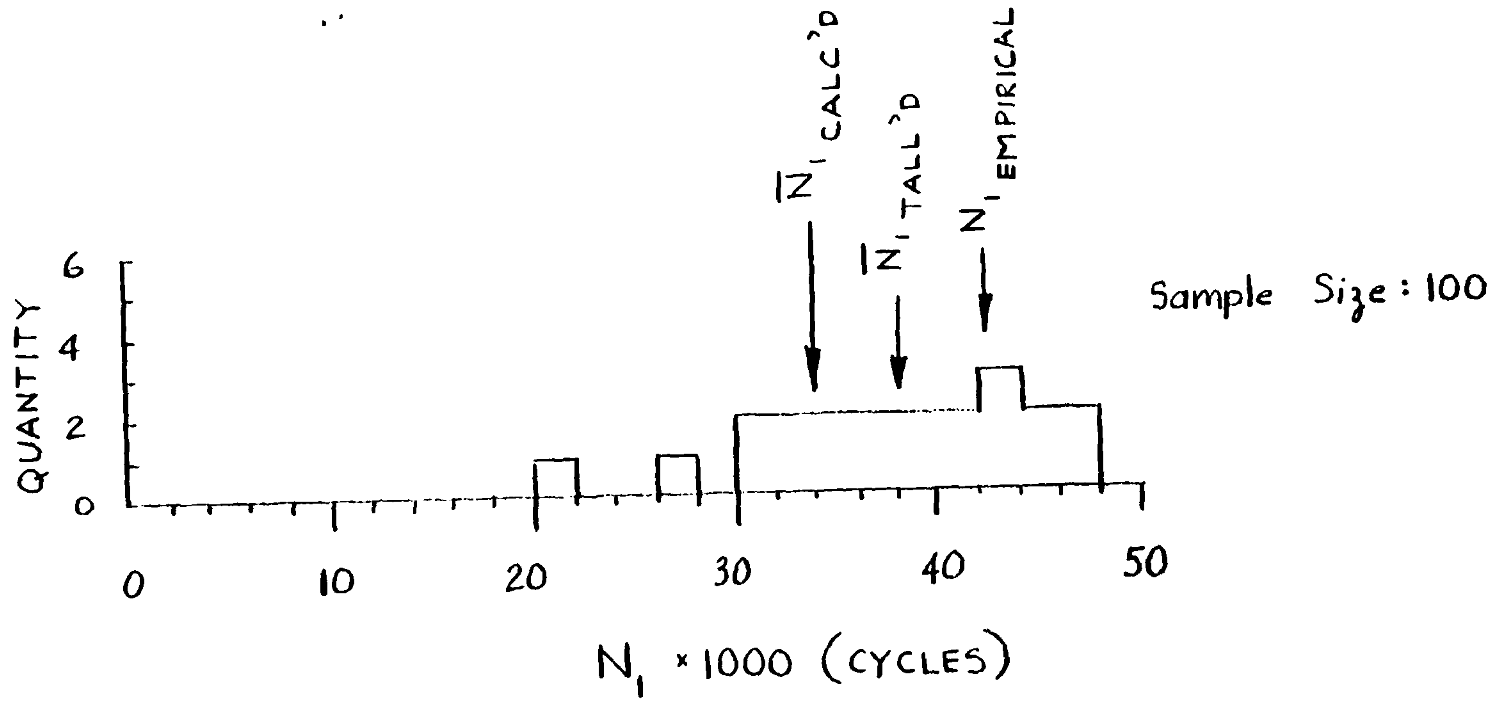
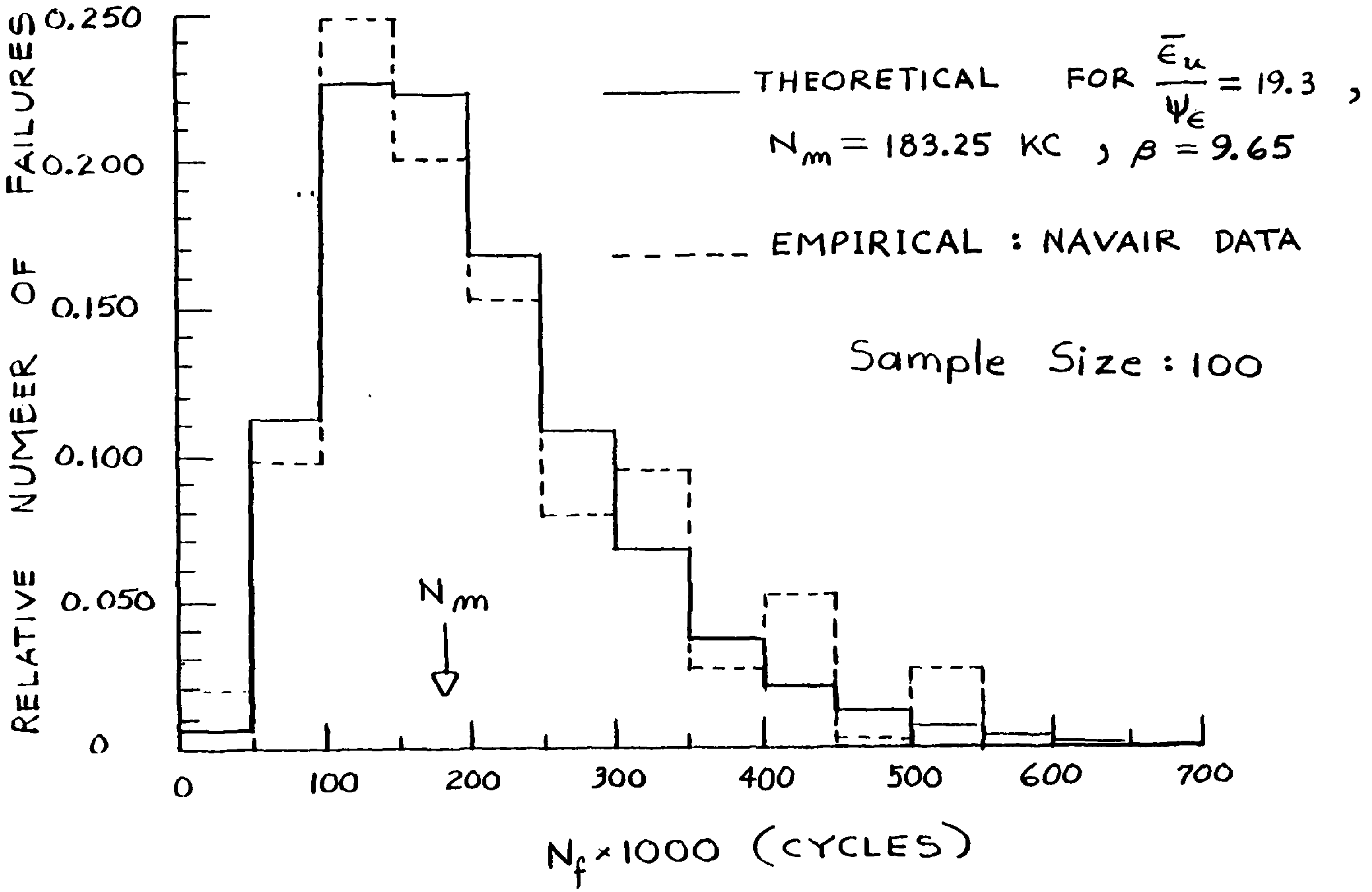


FIGURE 40
HISTOGRAM OF N_f : NAVAIR DATA



SYMBOLS

$\bar{A}$	material constant; true ultimate stress
b	fatigue strength exponent
c	fatigue ductility exponent
$\bar{C}$	constant of random fatigue curve
E	modulus of elasticity
erf(α)	error function of argument α
F(N)	probability of failure at N cycles
N	applied stress cycles
N_a, N_b	histogram bin width
N_f	number of stress cycles to failure
N_m	median stress cycles to failure; cycles to 50% failures
N_1	stress cycles to first failure
$\bar{N}_1$	average value of N_1
$N_{1_{MIN}}$	minimum value of N_1
N8	random variable
p(α)	probability density function of α
q	histogram quantity
rms	root mean square
S	total sample size
$\frac{\Delta S}{2}$	applied sinusoidal "engineering" stress amplitude
x,y,z r,h,v	general variables
α	general variable
β	fatigue curve slope parameter
$\Gamma(\alpha)$	gamma function of argument α

Δ	standard deviation of stress fatigue curve
Δ_ε	standard deviation of strain fatigue curve
δ	standard deviation of applied stress
δ_ε	standard deviation of applied strain
ε	applied strain amplitude (one-half applied strain range)
ε_μ	ultimate strain amplitude; ductility
$\bar{\varepsilon}$	average value of ε
$\bar{\varepsilon}_\mu$	average value of ε_μ
ε_f'	fatigue ductility coefficient
$\Delta\varepsilon$	applied strain range
$\Delta\varepsilon_e$	applied elastic strain range
$\Delta\varepsilon_p$	applied plastic strain range
§	correction factor
σ'_f	fatigue strength coefficient
$\bar{\sigma}$	average value of random rms stress
$\frac{\Delta\sigma}{2}$	applied sinusoidal "true" stress amplitude
σ_{N_1}	standard deviation of N_1
Ψ	resultant stress standard deviation
Ψ_ε	resultant strain standard deivation
Ψ_ε'	modified form of Ψ_ε

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APPENDIX A

DERIVATION OF $p(N_f)$

From equation (1)

$$2N_f = \frac{\varepsilon_\mu^{\beta}}{\varepsilon^{\beta}} \tag{A-1}$$

This is of the form $z = \frac{x}{y}$

where $x = \varepsilon_\mu^{\beta}$

$y = \varepsilon^{\beta}$

$z = 2N_f$

x and y are simultaneous random variables

Let ε_μ be a Gaussian random variable with average $\bar{\varepsilon}_\mu$ and standard deviation Δ_ε. From reference {9}

$$f(\varepsilon_\mu) = x = \varepsilon_\mu^{\beta} \tag{A-2}$$

$$\varepsilon_\mu = x^{1/\beta} \tag{A-3}$$

$$p(x) = \frac{p\ (\varepsilon_\mu)}{\left|\frac{d\ f(\varepsilon_\mu)}{d\ \varepsilon_\mu}\right|} \tag{A-4}$$

$$p(x) = \frac{p(\varepsilon_\mu)}{\beta\ \varepsilon_\mu^{\ \beta-1}} = \frac{\varepsilon_\mu^{\ 1-\beta}}{\beta}\ p(\varepsilon_\mu) \tag{A-5}$$

$$p(x) = \frac{x^{1/\beta-1}}{\beta\ \Delta_\varepsilon \sqrt{2\pi}}\ \exp\left[-\frac{\left\{x^{1/\beta} - \bar{\varepsilon}_\mu\right\}^2}{2\Delta_\varepsilon^{\ 2}}\right] \tag{A-6}$$

Similarly

$$p(y) = \frac{y^{1/\beta-1}}{\beta\ \delta_\varepsilon\ \ \ 2\pi}\ \exp\left[-\frac{\left\{y^{1/\beta} - \bar{\varepsilon}\right\}^2}{2\ \delta_\varepsilon^{\ 2}}\right] \tag{A-7}$$

$$p_{\underset{\sim}{x},\underset{\sim}{y}}(x,y) = p(x)\,p(y)$$

$$p_{\underset{\sim}{x},\underset{\sim}{y}}(x,y) = \frac{x^{1/\beta-1}\,y^{1/\beta-1}}{\beta^2\,\Delta_\varepsilon\,\delta_\varepsilon\,2\pi}\exp\left[-\frac{1}{2}\left\{\frac{(x^{1/\beta}-\bar{\varepsilon}_\mu)^2}{\Delta_\varepsilon^{\,2}}+\frac{(y^{1/\beta}-\bar{\varepsilon}_\mu)^2}{\delta_\varepsilon^{\,2}}\right\}\right] \tag{A-8}$$

$$p(z) = 2\int_0^\infty y\,p_{\underset{\sim}{x},\underset{\sim}{y}}(z\,y,\,y)\,dy \tag{A-9}$$

After much manipulation it can be shown that

$$p(N_f) = \frac{2^{1/\beta}\,N_f^{\,1/\beta-1}}{\beta\,\Delta_\varepsilon\,\delta_\varepsilon\,\pi}\left[\frac{1}{2}\sqrt{\frac{\pi}{r}}\,e^{-\frac{(h^2-r\,v)}{r}}\left\{2\,\bar{\varepsilon}\,\mathrm{erf}(\alpha_1)\right.\right.$$

$$\left.\left.+\frac{1}{\pi}\,e^{-h^2/r}-\left(\frac{2\,h}{r}\right)\mathrm{erf}\,(\alpha_2)\right\}\right] \tag{A-10}$$

$$\text{where } r = \frac{1}{2}\left[\frac{(2N_f)^{2/\beta}}{\Delta_\varepsilon^{\,2}}+\frac{1}{\delta_\varepsilon^{\,2}}\right]$$

$$h = -\left[\frac{(2N_f)^{1/\beta}\,\bar{\varepsilon}_\mu}{\Delta_\varepsilon^{\,2}}+\frac{\bar{\varepsilon}}{\delta_\varepsilon^{\,2}}\right]$$

$$v = \frac{1}{2}\left[\frac{\bar{\varepsilon}_\mu^{\,2}}{\Delta_\varepsilon^{\,2}}+\frac{\bar{\varepsilon}^{\,2}}{\delta_\varepsilon^{\,2}}\right]$$

$$\alpha_1 = \sqrt{2}\left[\sqrt{r}\;\bar{\varepsilon}+\frac{h}{\sqrt{r}}\right]$$

$$\alpha_2 = h\sqrt{\frac{2}{r}}$$

APPENDIX A (Cont'd)

$$\mathrm{erf}\,(\alpha) = \frac{1}{\sqrt{2\pi}} \int_0^{\alpha} e^{-y^2/2}\, dy$$

$\mathrm{erf}\,(0) = 0$; $\mathrm{erf}\,(\infty) = 0.5$

$\mathrm{erf}\,(-\alpha) = -\,\mathrm{erf}\,(\alpha)$

Equation (A-10) is the expression for $p(N_f)$ when ε_μ and ε are simultaneous random variables.

APPENDIX B

DERIVATION OF $\overline{N}_1$

$\overline{N}_1$ = average value of cycles to first failure

S = number of opportunities for failure

S = 10,000 for these cases

$$F(\overline{N}_1) = 0.5 + \text{erf}\ (\alpha);\ \text{erf}\ (\alpha) = -0.4999 \qquad \text{(B-1)}$$

$$\alpha = \text{erf}^{-1}\ (-0.4999) = -3.7195451$$

$$\alpha = \frac{\overline{\varepsilon}_\mu}{\Psi_\varepsilon}\left[\left(\frac{\overline{N}_1}{N_m}\right)^{1/\beta} - 1\right] = -3.7195451 \qquad \text{(B-2)}$$

Solving for $\overline{N}_1$

$$\overline{N}_1 = N_m \left[1 - \frac{3.7195451}{(\overline{\varepsilon}_\mu/_{\Psi_\varepsilon})}\right]^\beta \qquad \text{(B-3)}$$

$$N_{1_{MIN}} \cong \frac{1}{2}\left(\frac{\overline{\varepsilon}_\mu - 4.52\Delta_\varepsilon}{\overline{\varepsilon} + 4.52\delta_\varepsilon}\right)^\beta \qquad \text{(B-4)}$$

$$\sigma_{N_1} \cong (\overline{N}_1 - N_{1_{MIN}})/3 \qquad \text{(B-5)}$$